Changes in Employment and Wages in the United States, 1980–2019: Skills, Occupations, and Technology *

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October 17, 2026

Abstract

Changes in U.S. employment and wages since 1980 pose a joint puzzle: employment expanded at both ends of the occupational wage distribution, but relative wage gains were concentrated among high-skill workers and were larger still for managers. We construct a tractable model of skills, occupations, and sectors that distinguishes the forces behind these movements. Individuals differ in worker and managerial skills, and an additive occupational requirement governs comparative advantage among workers. We characterize equilibrium and derive conditions under which technological change affects worker assignment, wage differences among workers, managerial earnings relative to workers, and sectoral reallocation. In the 1980–2019 quantitative exercise, task-productivity changes account for much of employment reallocation but produce too little high-skill and managerial wage growth and too much managerial employment growth. Changes in requirements and skill supply help reconcile employment and wages. Occupational task characteristics support the distinction between productivity and effective-skill changes. Illustrative scenarios based on broad-AI and large-language-model exposure both yield higher abstract relative-wage growth but slower abstract employment growth than a continuation benchmark.

*The theoretical model in this paper was developed in conjunction with a project sponsored by PEDL and DFID, whose financial support (MRG 2356) we gratefully acknowledge. The paper benefited from comments and suggestions from many seminar and conference participants. We are grateful to Frederico Belo, Nancy Stokey, and Jon Willis, whose conference discussions helped greatly improve the paper. We also thank Sangmin Aum for outstanding research assistance. The usual disclaimer applies.

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1 Introduction

Job polarization, structural change, and rising wage inequality are well-documented features of the U.S. economy since 1980 (Autor et al., 2006, 2008; Herrendorf et al., 2014). But employment and wages changed differently across occupations. Employment expanded at both ends of the wage distribution, while relative wage gains were concentrated at the top. Low-skill jobs expanded without comparable relative wage gains; high-skill workers gained relative to other workers, and managers gained more still. These differences suggest that the expansion of jobs at the bottom and the rising rewards at the top operate through different margins. In this paper, we develop a framework that accounts for both employment and wages, allowing us to distinguish the roles of task productivity, occupational skill requirements, and the supply of skills. These channels need not have separate underlying causes: a common technological change can affect both the productivity of work and the skills used in it.

The distinction between managers and other high-paying occupations sharpens this question. Professionals and management-support workers gained both employment share and wages relative to manual workers. Managers experienced continued relative-wage growth even after their employment share peaked: between 2000 and 2019 their share fell from 14.0 to 13.0 percent, while their mean wage relative to manual workers rose from 2.51 to 3.31 (Table 1). Their employment share did rise over 1980–2019 as a whole, the period of our quantitative comparison. We therefore distinguish the return to worker skill from the scarcity and selection of managerial skill, while focusing the quantitative analysis on the long-run changes.

In our model, individuals are heterogeneous in two dimensions of skill—managerial skill and worker skill—based on which they choose to become a manager or a worker. Workers then select into occupations that differ in how much of their skill can be used in production. Managers combine occupation-level outputs to produce sectoral goods, and sectors differ in how intensively they use different occupations. The model separates task-specific productivity from occupational skill requirements and the supply of skills. Productivity determines output for given inputs, while requirements govern comparative advantage and the return to worker skill. Managerial selection simultaneously determines the supply of management, allowing us to study the wage premium of managers alongside wage differences among workers and occupational employment. Despite these different layers, the model remains tractable enough to characterize equilibrium, including unique relative service prices, and to derive analytical comparative statics.

A contribution of the model is a parsimonious specification of comparative advantage through occupational skill requirements. Instead of assigning different powers to worker skill across tasks, we subtract a requirement $\chi(j)$ from skill h, where j indexes tasks. Higher-skill workers are then relatively more productive in more demanding tasks: ordered requirements directly deliver log-supermodularity. The common linear skill term makes derivatives explicit and occupational boundaries simple to compute at given task prices. With discrete tasks, effective input depends only on employment and selected mean skill net of the requirement. The specification thus combines an economic interpretation of comparative advantage with convenient analytical and numerical implementation. Task productivity changes output for given effective inputs; requirements also change relative productivity across worker skills. We refer to changes that favor higher-skill workers in relative terms as *upskilling*, and derive sufficient conditions for this effect after equilibrium reassignment. In the quantitative model, manual work uses a constant effective input per worker, while nonmanual work uses skill net of the requirement. This distinction allows low-end employment to expand without the same direct skill-return channel that operates at the top.

Our theoretical analysis shows how these forces can have different implications for employment and wages. Under the stated local conditions, faster productivity growth among middle-skill tasks polarizes worker assignment and wage growth, and raises fixed-type managerial earnings relative to workers. A class of requirement changes instead raises the relative wages of higher-skill workers, and allows sufficiently

high-skill workers to gain relative to managers of given skill. Changes in relative managerial productivity or the supply of managerial skills provide a further force by changing the scarcity of management. Under additional skill-dependence conditions, this force also affects wage differences among workers. These effects need not have the same sign, and occupational mean wages also depend on who enters and leaves each occupation. The comparative statics distinguish the separate mechanisms; the quantitative exercise determines their combined magnitudes.

Armed with these results, we calibrate the model to the U.S. economy in 1980 and 2019. Our first quantitative result is that task-productivity changes account for much of the reallocation of employment. Manufacturing uses routine tasks more intensively than services, so sector-neutral productivity changes can also generate its declining employment share without exogenous sector-specific productivity growth. Second, unlike employment changes, wage changes are poorly explained by productivity growth alone: the model produces too much manual wage growth, too little abstract and managerial wage growth, and too much managerial employment growth. Changes in requirements and the skill distribution help reconcile these outcomes in the full model. Since wages and employment both discipline the calibration, the contribution of each force is assessed through the counterfactuals rather than through the wage fit alone.

We then relate the recovered changes to occupational task characteristics. Task-productivity growth is associated with routine-manual content and employment contraction, while changes in effective skill relative to mean skill are associated with nonroutine analytical content, educational attainment, and wage growth. Employment and wages enter the calibration; task content and educational attainment do not. The latter associations support an interpretation in which the activities associated with the movement of jobs differ from those associated with the changing rewards to skills.

The distinction is especially relevant for artificial intelligence (AI), whose occupational incidence need not resemble routine-task changes in the past. The 2010–2019 evidence associates the AI exposure measure of Felten et al. (2019) with wage growth after controlling for the preceding wage change, but finds no statistically significant employment-share association; other measures give different results. In two illustrative scenarios, using the broad-AI measure of Felten et al. (2019) and the task-level large language model (LLM) exposure measure of Eloundou et al. (2024), abstract employment grows more slowly than in the continuation benchmark, while abstract wages relative to the economy-wide mean grow faster. The LLM measure gives smaller abstract effects. Both measures give higher routine employment than continuation, while routine relative wages rise under LLM exposure and fall under broad AI. The regressions are descriptive and the scenarios are conditional projections, but both illustrate why employment exposure alone gives an incomplete account of a technology's distributional consequences.

The remaining applications distinguish occupational from sectoral employment objectives and examine a different direction of technological change. Taxing routine-task capital preserves some routine employment at an output cost. Subsidizing manufacturing also expands nonroutine jobs, so restoring manufacturing need not restore routine work. Work-from-home (WFH) and infectious-exposure scenarios redirect productivity growth across tasks while holding the continuation of skills common. These applications connect the historical mechanisms to new technologies and policies; they do not treat employment preservation as a welfare criterion.

Related literature Our model combines the assignment of heterogeneous skills to tasks in Costinot and Vogel (2010) with the occupational and sectoral structure of Stokey (2016). We extend this framework by distinguishing task productivity from occupational requirements and by introducing a separate managerial skill. The additive requirement parameterizes comparative advantage without task-specific powers on worker skill, and gives changes in relative returns an interpretation in terms of the skills used in production. Selection between management and worker occupations connects changes in skill prices to occupational mean earnings. Together, these features allow the same technological change to generate different wage

and employment responses at the bottom and top of the occupational distribution.

Occupational selection follows comparative advantage as in Roy (1951). Burstein et al. (2019) use quantitative assignment to account for between-group inequality through workforce composition, task demand, computerization and labor productivity. Our distinction between task productivity and requirements adds a margin through which technology changes the returns to skills, alongside selection into management. This distinction complements the broader account of technological change in Acemoglu et al. (2025). Automation and new-task creation in Acemoglu and Restrepo (2018, 2022) change the allocation of tasks between labor and capital; our exercises change productivity and requirements within a fixed set of tasks.

Autor and Thompson (2025) provide a close connection to the requirement channel. They show how changing the expertise required by a bundle of tasks affects both wages and access to an occupation. In our model, an additive requirement changes comparative advantage across worker skills, while selection into management links these worker outcomes to managerial earnings. The two approaches share the implication that a technology can change the rewards to an occupation by changing the skills used in it, as well as its productivity.

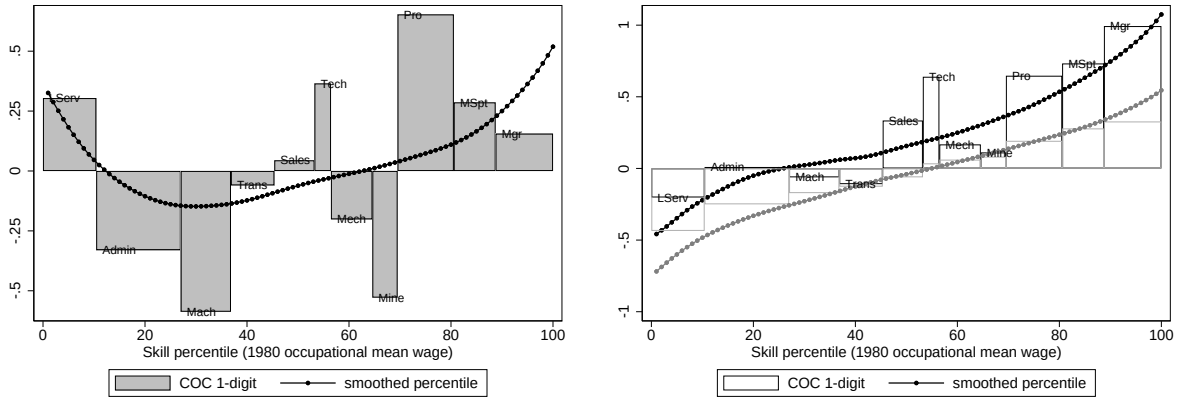
Althoff and Reichardt (2026) study AI in a dynamic task-based model with multidimensional skills, occupational choice and on-the-job skill accumulation. Their task-simplification channel lowers skill requirements and changes comparative advantage, providing a related application of the requirement margin. Our historical decomposition distinguishes changes in task productivity, requirements and skill supply, including selection into management; AI is one application of this framework.

Alonzo and Gallipoli (2026) also study occupational employment and earnings jointly, emphasizing how latent non-wage job values shift labor supply and affect equilibrium wages. We abstract from this non-wage margin and study how productive skills, occupational requirements, and selection between management and worker tasks change occupational labor supply. Task content and educational attainment provide evidence for interpreting the recovered changes. Joint wage and employment movements can therefore reflect both productive and non-wage sources of comparative advantage.

The managerial technology extends the span-of-control setting of Lucas (1978). Managers choose the composition of worker tasks and the effective skills used in them, with a constant-elasticity-of-substitution (CES) relationship between managerial and worker inputs. This separates the managerial wage premium from wage differences among workers. In a knowledge-hierarchy setting, Ide and Talamàs (2025) show that AI's distributional effects depend on whether it operates autonomously or assists humans. Our AI exercises hold the organizational technology fixed and study changes in requirements and occupational selection.

Goos et al. (2014) explain employment polarization within and between industries through task-specific capital costs and offshoring. Our equilibrium also determines the wage and selection responses, and aggregate output disciplines the size of the recovered productivity changes. Occupation-specific technological change generates structural transformation in Duernecker and Herrendorf (2022). Related accounts connect skill demand, polarization and structural change, including Buera and Kaboski (2012), Buera et al. (2015), Bárány and Siegel (2018), and Bárány and Siegel (2020). Our sectoral productivity differences arise through occupational choices and task composition, connecting production-driven structural change as in Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) to the changing rewards to worker and managerial skills. Structural change is one consequence of the joint occupational mechanism.

Section 2 documents wage and employment changes. Section 3 develops the model and analytical comparisons. Sections 4 and 5 calibrate the 1980–2019 economy, compare the mechanisms, and relate the recovered changes to occupational characteristics. Section 6 applies the framework to AI, robot taxes, manufacturing subsidies, and WFH. Section 7 concludes.



(a) Employment Polarization

(b) Occupational Average Wage, 1980 and 2019

Fig. 1: Employment and Wage Change by Occupation, 1980–2019.
Source: U.S. Census and ACS. Occupations are ranked by their 1980 mean wage. Bars show 11 broad groups; the smoothed curves use 310 detailed occupations. Bar widths equal the broad groups' 1980 employment shares. Panel (a) shows employment-share changes in percentage points per employment percentile. Panel (b) shows log mean wages relative to the economy-wide mean in 1980, with gray bars for 1980 and black bars for 2019. See Appendix A.

2 Facts

We document wage changes alongside employment reallocation across occupations and sectors. The central wage facts are the larger gains at the top of the worker distribution and the still larger gains for managers. Employment polarization and structural change provide the accompanying quantity changes, but their timing and incidence do not fully explain the wage pattern. We then relate both margins to task content and educational attainment.

2.1 Occupational Employment and Wages

Our occupational data are from the U.S. Census and American Community Survey (ACS), for 1980, 1990, 2000, 2010, and 2019. Following Autor and Dorn (2013), we restrict the sample to non-farm workers aged 16–65. Figure 1 extends their employment and wage comparisons through 2019. Occupations are sorted into employment percentiles along the x-axis, with skill proxied by each occupation's mean hourly wage in 1980. We use the three-digit `occ1990dd` codes in Dorn (2009), which harmonize the `occ1990` classification in Meyer and Osborne (2005). This gives 310 occupations with positive employment in all five sample years.[1] Employment is measured by weeks worked times usual weekly hours.

The data are presented in two ways. First, following Autor and Dorn (2013), each dot in Figure 1 represents one percent of employment in 1980. The y-axis in Panel (a) measures each skill percentile's employment change from 1980 to 2019 in percentage points, and in Panel (b) its log mean wage relative to the economy-wide mean in 1980. The changes are smoothed by percentiles across neighboring occupations using a locally weighted smoothing regression. Despite the Great Recession, the long-run patterns through 2019 are similar to those in their study: employment has shifted from the middle toward both lower- and higher-skill jobs. Likewise, wages have risen the least in the middle, and much more at the top.

Second, we group all occupations into 11 broad categories, roughly (but not exactly) corresponding to the one-digit Census Occupation Codes (COC). These groups are ordered by the mean wage of each

[1]The number of occupation codes obtained by harmonizing the censuses with the 2019 ACS differs slightly from Dorn (2009), who harmonized census occupation codes with the 2005 ACS.

	Employment share (%)			Mean wage / manual wage		
	1980	2000	2019	1980	2000	2019
Professionals	11.0	14.5	18.2	1.87	1.96	2.34
Management support	8.2	8.5	10.5	2.04	2.06	2.55
Managers	11.3	14.0	13.0	2.15	2.51	3.31

Table 1: Managers and Other High-Paying Occupations

Source: 1980 and 2000 Census and 2019 ACS, using the occupational aggregates underlying Figures 1 and 2. Employment shares use annual hours and Census sampling weights and include the self-employed. Mean hourly wages exclude self-employment income and are divided by the low-skill-service (manual) mean wage in the same year. See Appendix A for the sample and occupation definitions.

broadly defined group. To represent the groups in skill percentiles, the width of each group along the x-axis is set equal to its 1980 employment share.[2] In Figure 1(a), the percentage point change of a group's employment share is represented by the area of the bar,[3] while the bars in Panel (b) show each group's mean wage in 1980 and 2019. Except for sales and technicians, the patterns of polarization emphasized by Acemoglu and Autor (2011) and Autor and Dorn (2013) remain intact through 2019.[4,5]

2.2 Managers and Other High-Paying Occupations

Managers earned the highest wages in 1980 and experienced the largest subsequent wage gains, yet their employment share increased only modestly in the aggregate and barely at all in services. The relevant comparison is not only with workers as a whole, but also with other high-paying occupations. Table 1 separates professionals and management support—the abstract-worker groups in our quantitative model—from managers. "Management support" is a worker occupation group, distinct from the managerial category.

Both abstract-worker groups gained employment share and relative wages between 1980 and 2019, and again between 2000 and 2019. Management combines a much larger relative-wage gain with a smaller long-run employment increase and a decline after 2000. High-paying occupations therefore exhibit two distinct wage–employment patterns. The expansion of high-skill worker occupations and the different managerial pattern supply two margins against which to assess explanations based on productivity and skills. The contrast motivates modeling managerial services and their endogenous supply separately.

In our model and quantitative analysis, we treat managers as a special occupation that organizes all other occupations. While many studies emphasize the organization of production (Garicano and Rossi-Hansberg, 2006), others focus on CEOs of publicly traded companies (Tervio, 2008; Gabaix and Landier, 2008) or certain industries (Caliendo et al., 2015). We treat managers as a broader group including CEOs, middle managers, and the self-employed, and also connect them to a notion of establishments.[6]

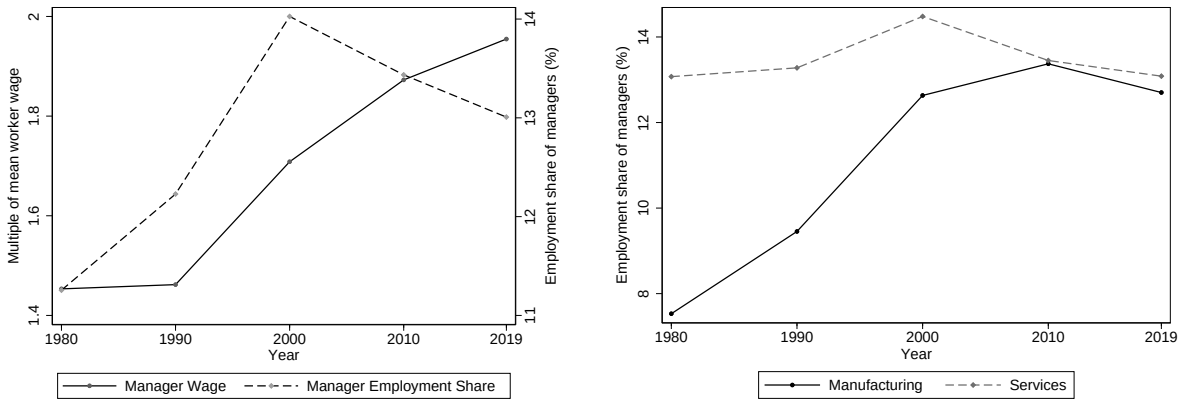
[2] See Appendix A for more details on wage, employment, and occupation definitions. The three-digit occupations in each group do not necessarily correspond to those used for the smooth graphs by percentile.

[3] By construction, the area of all bars must add up to 0. In theory, the smoothed graph should also integrate to 0, but does not due to the locally weighted regression errors.

[4] The occupation groups form the basis for the calibration in Section 4. Appendix Table F.1 reports the 1980 and 2019 relative-wage targets.

[5] Technicians include software engineers and programmers, paralegals, and health technicians, whose employment grew rapidly during this period. Occupations in this group attenuate the middle-skill employment decline in the smoothed graphs. But technicians comprise a very small fraction of employment throughout the sample period, and most of their increase occurs in services rather than manufacturing (Figure 3(b)).

[6] Appendix Figure A.1 shows the employment shares of the managerial and self-employed categories used in this definition. Employment includes the self-employed; the wage measure uses labor income and excludes self-employment income, as explained in Appendix A.



(a) Relative Employment and Wages

(b) Manager Employment Share

Fig. 2: Managers vs Workers

Source: U.S. Census and ACS. Left: Relative wage and employment share of managers in aggregate. Right: Employment share of managers within manufacturing and services. See Appendix A for the definition of managers and Figure A.1 for a detailed breakdown of the manager group.

Previous papers document rapid growth in CEO wages relative to the median worker's, and Figure 2(a) shows that even with our broader definition, the "manager wage premium" over all other workers rose from 45 percent in 1980 to more than 95 percent in 2019. At the same time, the employment share of managers also rose from 11 to 14 percent between 1980 and 2000, but then dropped back to 13 percent from 2000 to 2019.

The rise in the within-sector managerial employment share was pronounced only in manufacturing. Figure 2(b) shows that when separated by sector, managers' employment share rose by more than 4 percentage points in manufacturing but barely at all in services. Appendix Figure F.10(a) shows that the shift from manufacturing to services was much smaller among managers than among other workers. [7]

2.3 Sectoral Reallocation

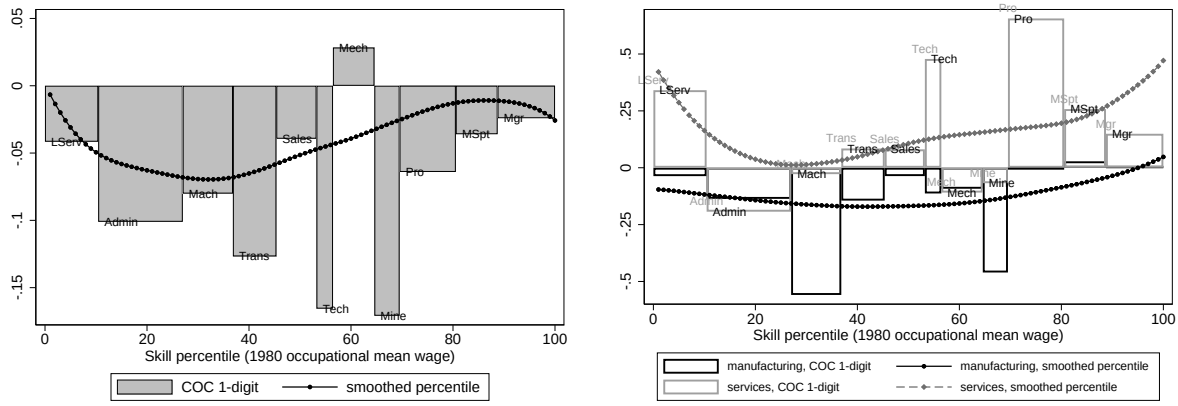
Figure 4 shows nominal value-added and employment share trends of three broadly defined sectors: agriculture, manufacturing and services, from 1947 to 2019. Following convention, e.g. Herrendorf et al. (2014), "manufacturing" is the aggregation of manufacturing, mining and construction, and "services" the sum of all service and government sectors. The data are from the National Accounts published by the Bureau of Economic Analysis (BEA). In particular, employment is based on National Income and Product Accounts (NIPA) Table 6 (persons involved in production), counted in terms of full-time equivalent workers.[8]

Agriculture accounts for a small share of the U.S. economy over our period of study. For the remainder of the paper, we exclude agriculture and compute all moments for manufacturing and services.

Structural change—the shifting of GDP and employment from manufacturing to services—exhibits a smooth trend starting from at least the 1970s, as noted in Herrendorf et al. (2014). The nominal output and employment shares move broadly together. This comparison does not identify sectoral labor income

[7] Appendix Figure F.10(b) plots the wage ratio between managers working in the manufacturing sector and in the services, as well as the same ratio for all other workers. Both manufacturing-to-services wage ratios change by less than 0.10, and we do not seek to explain why wages differ across sectors within the same occupation. The different wage patterns for managers and workers provide an additional dimension against which to assess the model.

[8] Computing employment shares from the decennial census yields more or less the same trend.



(a) Change in Manufacturing Employment Share

(b) Employment Polarization by Sector

Fig. 3: Employment Change by Sector and Occupation, 1980–2019.

Source: U.S. Census and ACS. Panel (a): Change in the manufacturing employment share within each occupation. Panel (b): Change in occupational employment by sector. Occupations are ranked by their 1980 mean wage for 11 one-digit groups and smoothed across 310 three-digit groups. The horizontal axis measures 1980 employment percentiles. Changes are measured over 1980–2019; in Panel (b), units are percentage points per percentile. Further details are in Appendix A.

shares, which also depend on earnings per worker. We set the capital income share separately in Section 4.2.

Appendix Figure F.15 shows total factor productivity (TFP) in manufacturing and services from 1948 using sector-specific average capital income shares (0.332 in manufacturing and 0.365 in services).[9] Note that manufacturing's TFP relative to services was more or less constant from the mid-1960s to early 1980s, after which it quickly increased.[10] In our quantitative analysis, we relate this to faster task-specific technological progress among middle-skill jobs, which began to show a distinct declining trend around the same time (Autor and Dorn, 2013).

Figure 4 also shows a slowdown in structural change in the 2010s, especially in employment shares, alongside slower relative manufacturing TFP growth in Figure F.15. Our analysis ends in 2019, before the Covid-19 pandemic, and concerns these longer-run changes.

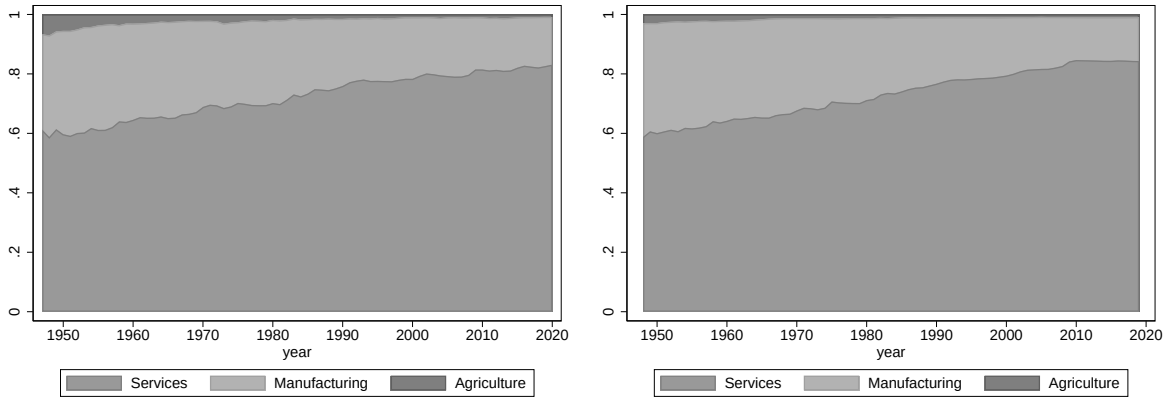
We next examine how occupational polarization and sectoral reallocation are related. Figure 3 returns to the Census and ACS data and separates employment changes by occupation and sector. In Panel (a), occupations are ordered along the x-axis in the same way as we did in Figure 1. For each occupation, we compute the employment share of manufacturing in 1980 and 2019, and plot the difference.[11] The bars measure the percentage point change in the share of manufacturing employment within each COC occupation group, and the dots the smoothed percentage point change for each skill percentile. The entire plot is negative, which represents structural change from manufacturing to services. More important, manufacturing shrank the most in the middle (except mechanics). In Panel (b) we plot the same changes as in Figure 1(a), but separate manufacturing (in dark) and services (in light).[12] Manufacturing lost

[9]This is the longest time period allowed by the industry accounts, which we need to compute real GDP and capital at the sector level. Herrendorf et al. (2015) argue that Cobb-Douglas sectoral production functions with equal capital income shares can quantitatively capture the effect of differentially evolving sectoral TFPs. See Section 4.2 for details.

[10]Herrendorf et al. (2014) also note that manufacturing's relative TFP has grown quicker than services post 1970s, but that such trends may not be stable over a longer horizon.

[11]Appendix Figure F.8(a) shows the share of manufacturing employment within each occupation in 1980, which shows manufacturing's reliance on middle-skill jobs.

[12]The area of all bars for one sector represents structural change, while adding them across both yields 0. Similarly, the integrals of the smoothed graphs should sum to 0, modulo locally weighted regression errors.



(a) Nominal Value-added Shares

(b) Sectoral Employment Shares

Fig. 4: Sectoral Output and Employment Shares, 1947–2019.
Source: BEA NIPA accounts. Manufacturing combines manufacturing, mining, and construction; services combines service and government sectors. Employment is measured in full-time equivalents. See Section 4.2.

employment across all jobs (except managerial support). The loss is more pronounced in the middle, especially machine operators and miners. In contrast, services gained employment overall, but mostly among occupation-skill percentiles in the far left and right (e.g., low-skill services and professionals, respectively).

Polarization occurs within both sectors, so aggregate polarization is not solely an accounting consequence of employment shifting between sectors with fixed occupational compositions. Task-specific forces can generate these patterns, as can sectoral shocks that change occupational composition. Moreover, as shown later in Figure 9, within-sector polarization was faster in manufacturing than in services: Between 1980 and 2019, the share of middle-skill jobs out of all manufacturing jobs fell by 13 percentage points, while the corresponding number in services was only 7 percentage points.

2.4 Task Content and Education

Changes in the education and skill composition of the workforce have long been central to explanations of employment and wage changes (Katz and Murphy, 1992; Acemoglu and Autor, 2011). The routinization literature relates employment shifts to technologies that replace repetitive tasks, using the Dictionary of Occupational Titles (DOT) or Occupational Information Network (O*NET) measures of occupational content (Autor et al., 2003; Acemoglu and Autor, 2011; Autor and Dorn, 2013). Complementary evidence comes from occupational software and robot exposure (Webb, 2020) and observed industrial robot adoption (Acemoglu and Restrepo, 2020). AI studies use exposure measures and vacancy data to examine another direction of technological change (Felten et al., 2019; Webb, 2020; Acemoglu et al., 2022).

Tables 2–3 show different task-content associations for employment and wages. Routine-manual content is associated with employment declines, while nonroutine analytical content is associated with wage growth. In both tables, we consider consolidated O*NET measures from Acemoglu and Autor (2011), robot and software exposure indices constructed by Webb (2020), and the change in the college share of workers in each occupation between 1980 and 2019 as explanatory variables. We standardize the task and exposure indices to have mean zero and unit standard deviation.

The O*NET measures considered are “Nonroutine-Cognitive Analytical”, “Nonroutine-Cognitive Interpersonal”, “Routine-Cognitive”, “Routine-Manual”, and “Nonroutine-Manual Physical.” When all

	(1)	(2)	(3)	(4)	(5)	(6)
NR-Cog Anal	0.128					
	(0.237)					
NR-Cog Pers	0.258					
	(0.255)					
R-Cognitive	-0.035					
	(0.128)					
R-Manual	-0.251	-0.601**		-0.555**		-0.589**
	(0.134)	(0.150)		(0.160)		(0.119)
NR-Man Phys	-0.204					
	(0.140)					
Robot (Webb)			-0.487*	-0.071		
			(0.227)	(0.138)		
Software (Webb)			0.087			
			(0.209)			
ΔCollege					2.761	0.268
					(1.982)	(1.839)
Constant	0.609**	0.609**	0.609**	0.609**	0.297	0.579**
	(0.128)	(0.129)	(0.131)	(0.129)	(0.204)	(0.210)
R^2	0.089	0.067	0.035	0.068	0.015	0.067
Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$.						

Table 2: Routinization and Employment

The dependent variable is the change in the employment share of three-digit COC occupations from 1980 to 2019. The first five independent variables are consolidated O*NET measures from Acemoglu and Autor (2011), while the next two are taken from Webb (2020). ΔCollege is the change in the share of college workers in each three-digit occupation.

measures are included as independent variables in Table 2, none is individually statistically significant for the change in the three-digit occupation employment share. But when we only include as a regressor the routine-manual task content, it has significant explanatory power. So does Webb's robot measure, but it becomes insignificant once we also include jobs' routine-manual content.

In Table 3, nonroutine-cognitive analytical task content is positively associated with wage growth even when the other O*NET measures are included. Nonroutine-manual physical content and Webb's robot measure have negative coefficients. Wage growth is also positively associated with the rise in an occupation's college-educated share.

The contrast between Tables 2 and 3 motivates distinguishing productivity and skill channels in the model. Routine-task measures are associated with employment reallocation, while analytical content and educational attainment help account for wage growth. These are descriptive relationships, rather than causal estimates of technological change. Section 6.1 asks whether the same relationships extend to 2010–2019 and examines the distinct wage and employment associations of AI exposure.

3 Model

Figure 5 summarizes the model. Individuals choose between management and worker tasks, and managers combine task outputs to produce sectoral goods. Occupational choice and production jointly determine employment and wages. We first characterize equilibrium, then derive comparative statics under the additional conditions stated for each result. Proofs are in the appendix.

	(1)	(2)	(3)	(4)	(5)	(6)
NR-Cog Anal	0.087**	0.097**		0.089**		0.073**
	(0.015)	(0.013)		(0.014)		(0.013)
NR-Cog Pers	0.005					
	(0.017)					
R-Cognitive	0.019					
	(0.011)					
R-Manual	-0.036*					
	(0.017)					
NR-Man Phys	-0.039**	-0.060**		-0.033*		
	(0.014)	(0.012)		(0.016)		
Robot (Webb)			-0.134**	-0.044**		-0.036**
			(0.016)	(0.016)		(0.012)
Software (Webb)			0.041*			
			(0.016)			
ΔCollege					1.316**	0.913**
					(0.117)	(0.151)
Constant	0.270**	0.270**	0.270**	0.270**	0.121**	0.167**
	(0.011)	(0.011)	(0.012)	(0.011)	(0.017)	(0.020)
R^2	0.344	0.333	0.231	0.350	0.336	0.468
Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$.						

Table 3: Skills, Task Content, and Wage Growth

The dependent variable is wage growth in three-digit COC occupations from 1980 to 2019. The first five independent variables are consolidated O*NET measures from Acemoglu and Autor (2011), while the next two are taken from Webb (2020). ΔCollege is the change in the share of college workers in each three-digit occupation.

3.1 Technology

There is a continuum of individuals endowed with two types of skill, $(h, z) \in \mathcal{H} \times \mathcal{Z} \subset \mathbb{R}^2_+$. Worker human capital, h, is used to produce worker tasks. Managerial talent, z, is a skill for organizing tasks. Worker skill h governs comparative advantage across worker tasks, while managerial skill z governs productivity in management. Individuals compare the earnings from these two uses of skill, so managerial supply depends on selection as well as the joint skill distribution. This permits the reward to organizing production to rise differently from the rewards to performing high-skill worker tasks. Without loss of generality, we assume that the mass of individuals is 1, with associated cumulative distribution function $\mu(h, z)$.

There are two sectors $i \in \{m, s\}$.[13] In each sector, goods are produced by teams. A team is led by a manager who uses his managerial skill and physical capital to organize a continuum of worker tasks $j \in \mathcal{J} = [0, J]$, where J is finite.

We refer to the manager's job as "task z," which is qualitatively different (in a hierarchical sense) from tasks $j \in \mathcal{J}$, which are horizontally differentiated among workers. Each worker task requires both physical and worker human capital, and their allocations are decided by the manager. Specifically, we assume that

$$y_i(z) = \left[\eta_i^{\frac{1}{\omega}} x_z(z)^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} x_h(z)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{1a}$$

$$x_z(z) = M(z) k_{iz}(z)^{\alpha} z^{1-\alpha}, \qquad x_h(z) = \left[\int_{j=0}^{J} \nu_i(j)^{\frac{1}{\sigma}} \tau_i(j; z)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{1b}$$

[13] In our application, the two sectors indexed by m and s stand for "manufacturing" and "services," respectively. However, the analysis can be extended to any finite number N of sectors.

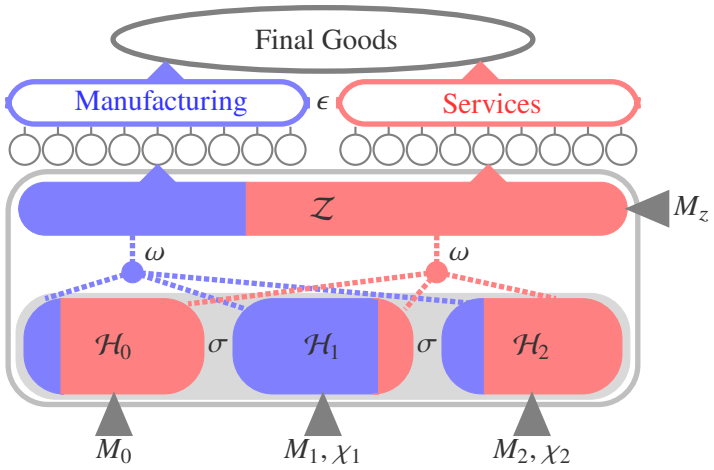


Fig. 5: Model

Individuals choose management or worker tasks according to managerial skill z and worker skill h. While the model has a continuum of skills and tasks, in the figure we depict the latter as three discrete groups. Tasks are combined with elasticity σ, and worker output is combined with managerial output with elasticity ω. Each team is led by a manager, and the collection of team output is sectoral output. Sectoral outputs form final output with elasticity ϵ. The shaded areas show how intensively each sector uses each task. The sets $\mathcal{Z}$ and $\mathcal{H}_j$ in the drawing denote managers and workers in task group j; M_j and χ_j denote task productivity and the occupational requirement.

$$\tau_i(j;z) = M(j)k_{ih}(j;z)^{\alpha}\left[\int_{\mathrm{h}_i(j;z)} b(h,j)d\mu\right]^{1-\alpha}, \tag{1c}$$

where $\alpha \in (0,1)$ is the common capital share, σ is the elasticity across worker tasks, and ω is the elasticity between worker and managerial output. The task-productivity parameters are $M(j)$ for workers and $M(z)$ for managers; the argument z in $M(z)$ labels the managerial task, so this parameter is common to managers of all skills. The weights satisfy $\eta_i \in (0,1)$, $\nu_i(j) \geq 0$, and $\sum_i \nu_i(j) > 0$ almost everywhere, and $\int_j \nu_i(j)dj = 1$.[14] Sectors differ in how intensively they use each task in production, as captured by the sector-specific weights ν_i and η_i. The quantity $\tau_i(j;z)$ is the amount of task j output produced for a manager of skill z in sector i. This manager uses physical capital $k_{iz}(z)$ for himself, and allocates capital $k_{ih}(j;z)$ and a set of workers $\mathrm{h}_i(j;z)$ to task j. The function $b(h,j)$ is the productivity of a worker with human capital h assigned to task j.[15] Without loss of generality, normalize $b(0,0) = 1$. This fixes productivity at one skill–task pair; it does not make $b(h,0)$ constant in h. The general analytical model permits skill-dependent productivity even in the lowest task. For unique sorted worker assignment, assume that

Assumption 1 (Log-supermodularity) *The function $b : \mathcal{H} \times \mathcal{J} \mapsto \mathbb{R}_+$ is strictly log-supermodular. That is, for all $h' > h$ and $j' > j$:*

$$\log b(h',j') + \log b(h,j) > \log b(h',j) + \log b(h,j'). \tag{2}$$

Assumption 1 ensures that high-h workers sort into high-j tasks in equilibrium. Integrating $b(h,j)$ over h of the workers in the set $\mathrm{h}_i(j;z)$ yields the total productivity of all workers assigned to task j by a

[14] A useful mnemonic is index i for industry (sector) and j for job (task).

[15] Except for Costinot and Vogel (2010), most task-based models assume that either tasks or skills are a continuum, but not both (Acemoglu and Autor, 2011; Stokey, 2016). The worker side of our model is similar to theirs, but includes capital as a production factor and extends to multiple sectors. We add two-dimensional skills and distinguish managers from workers, allowing managerial selection to interact with assignment across worker tasks.

manager of skill z in sector i.

Skill Loadings and Occupational Requirements The economic content of log-supermodularity can be seen in a simple example with different skill loadings. At fixed positive task prices p_j, suppose earnings in task j are $y_j(h) = p_j h^{\lambda_j}$ for $h > 0$. For two tasks $j' > j$,

$$\log \frac{y_{j'}(h)}{y_j(h)} = \log \frac{p_{j'}}{p_j} + (\lambda_{j'} - \lambda_j) \log h.$$

With a common loading, skill raises earnings in both tasks by the same proportion and does not affect their relative return. If $\lambda_{j'} > \lambda_j$, higher-skill workers instead have a comparative advantage in task j'.

Our requirement specification gives this comparative advantage an interpretation in terms of the skills used by a task. In the additive benchmark $b(h, j) = h - \chi(j)$, $\chi(j)$ is the task requirement measured relative to a common baseline capacity; its level can therefore be negative. A higher requirement reduces the skill available for production, proportionately more for a low-skill worker. On the domain where both effective skills are positive,

$$\frac{\partial}{\partial h} \log \frac{p_{j'}[h - \chi(j')]}{p_j[h - \chi(j)]} = \frac{\chi(j') - \chi(j)}{[h - \chi(j')][h - \chi(j)]} > 0 \quad \text{if } \chi(j') > \chi(j).$$

Thus ordered requirements implement log-supermodularity without assigning a different power to each task. The additive form has both interpretive and practical advantages. Earnings $p_j[h - \chi(j)]$ are affine in skill at given task prices, so boundaries between active occupations solve linear indifference equations. Skill derivatives are explicit. In the discrete model, effective task input is employment times selected mean skill net of the requirement, avoiding separate power moments for different tasks. These properties simplify implementation, while prices and managerial selection remain jointly determined in equilibrium. Requirement changes that favor higher-skill workers in relative terms are upskilling. Log-supermodularity describes the ordering at a given date; upskilling describes a change in relative returns. Proposition 4 states sufficient conditions for the equilibrium effects of such a change. The general model allows other log-supermodular technologies, while the quantitative specification in Section 4 uses additive requirements for nonmanual tasks and a separate flat manual task.

Task productivities are sector-neutral. We use $\mathcal{J}^z \equiv \mathcal{J} \cup \{z\}$ to denote the worker tasks together with management.

Let $\mathcal{Z}_i$ denote the set of individuals working as managers in sector i. Aggregating over the output from all managers in sector i yields sectoral output

$$Y_i = A_i \int_{\mathcal{Z}_i} y_i(z) d\mu, \quad i \in \{m, s\}, \tag{3}$$

where A_i is an exogenous, sector-level productivity parameter. Final goods are produced by combining output from both sectors according to a CES aggregator:

$$Y = \left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $\gamma_m + \gamma_s = 1$. The equilibrium result allows any finite positive ϵ; the quantitative analysis uses $\epsilon < 1$.[16]

[16]The estimated ϵ between the manufacturing and the service sectors (broadly defined) is close to 0, as we show in Section 4.2. As usual, (4) can be viewed as a representative consumer's preferences over the two sources of output.

Assumption 2 (Technology and task weights) *The common capital share satisfies* $0 < \alpha < 1$*, capital supply is* $K > 0$*, and* σ, ω, ϵ *are finite and positive. Sector and managerial productivities are positive and finite. Worker-task productivity* $M(j)$ *is positive, finite, and measurable almost everywhere, with* $0 < \int \nu_i M^{\sigma-1} < \infty$ *for* $\sigma \neq 1$*; at* $\sigma = 1$*, require* $\int \nu_i |\log M| < \infty$*. The weights satisfy* $\gamma_i > 0$*,* $\sum_i \gamma_i = 1$*,* $0 < \eta_i < 1$*, and* $\nu_i(j) \geq 0$ *with* $\int \nu_i(j)dj = 1$ *and* $\sum_i \nu_i(j) > 0$ *almost everywhere. Zero-weight tasks are omitted from a sector's aggregator. The function* b *is positive and measurable, with* $0 < \mathbb{E}b(h, 0) < \infty$*. Its relative-task productivity* $r_b(h, j) = b(h, j)/b(h, 0)$ *is uniformly bounded above and away from zero, and is uniformly continuous in* j*, uniformly over* h*. These bounds follow automatically from a positive continuous* b *on a compact worker/task domain, but also allow unbounded worker skills with finite productivity moments. Unit elasticities use Cobb–Douglas limits.*

3.2 Planner's Problem

Since our model has no frictions, the planner's allocation coincides with a competitive equilibrium allocation.[17] A planner allocates aggregate capital K and all individuals into sectors $i \in \{m, s\}$ and tasks $j \in \mathcal{J}^z$. Formally, define $h_i(j; z)$ as the amount of effective worker skill the planner allocates to task j in sector i under a manager with skill z. Also define $l_{ih}(s, j)$ as the number of individuals with skill $s = (h, z)$ that the planner assigns to task j, and $l_{iz}(s)$ the number of individuals with skill s the planner assigns as managers, in sector i. Then the planner's problem is to choose factor allocation rules $\{k_{iz}(z), k_{ih}(j; z), h_i(j; z)\}$ and assignment rules $\{l_{ih}(s, j), l_{iz}(s)\}$ to maximize current output (4) subject to

$$
\begin{aligned}
&Y_i = A_i \int y_i(z) l_{iz}(s) ds, \quad \forall i \in \{m, s\}, \\
&K = K_m + K_s \equiv \sum_{i \in \{m,s\}} \int \left\{ \left[k_{iz}(z) + \int_j k_{ih}(j; z) dj \right] \cdot l_{iz}(s) \right\} ds \qquad (5) \\
&H_i(j) \equiv \int h_i(j; z) \cdot l_{iz}(s) ds = \int b(h, j) l_{ih}(s, j) ds \quad \forall i \text{ and } \forall j \in \mathcal{J}, \\
&d\mu(s) = \sum_{i \in \{m,s\}} \left[\int l_{ih}(s, j) dj + l_{iz}(s) \right] ds, \quad \forall s \in \mathcal{H} \times \mathcal{Z}, \qquad (6)
\end{aligned}
$$

where K_i is the capital allocated to sector i, and $H_i(j)$ the total productivity of workers allocated to task j in sector i. These density-form expressions describe the regular case. For skill distributions with atoms or singular parts, type allocations are measures adding up to μ; tied types can be split across activities. The price proof in Appendix C.4 covers that general case.

For the equilibrium result we impose the following skill regularity:

Assumption 3 *Worker skill may have bounded or unbounded nonnegative support, and managerial skill satisfies* $0 < \mathbb{E}z < \infty$.

Let $G(h)$ be the marginal distribution of worker skill and $F(z \mid h)$ the conditional distribution of managerial skill. Then

$$
\mu(\tilde{h}, \tilde{z}) = \int_{[0,\tilde{h}]} F(\tilde{z} \mid h) dG(h).
$$

When densities exist, write them as $g(h)$ and $f(z \mid h)$. Let $w_h(h)$ denote worker earnings at skill h and w_z the price per unit of managerial skill. Individuals choose worker tasks when $z < w_h(h)/w_z$. For

[17] We thank Nancy Stokey for suggesting the competitive-equilibrium approach, presented in Appendix B. Solving the planner's problem avoids the intermediate task prices introduced by nested production and capital.

unique sorted worker assignment, the selected worker-skill measure $F(w_h(h)/w_z \mid h)dG(h)$ must be atomless; atomlessness of G suffices. Unique worker–manager choice additionally requires zero joint probability on the indifference curve. Conditional managerial atomlessness suffices, while atoms away from the cutoffs are permitted. Existence and uniqueness of service prices require neither a density nor common skill support. The differential characterization below concerns a regular interior equilibrium on a compact worker interval $[0, h_M]$: b is twice continuously differentiable, g and composite task-cost weights $\nu_i(j)M(j)^{\sigma-1}$ are continuous and positive, and worker participation $F(\tilde{z}(h) \mid h)$ is continuous and positive along the selection curve $\tilde{z}(h) = w_h(h)/w_z$. Density smoothness is needed only where the comparative-static proofs differentiate selected supplies. These regularity conditions are not assumptions for the service-price existence result.

The optimal factor allocation rules across managers, $\{k_{iz}(z), k_{ih}(j;z), h_i(j;z)\}$, are straightforward: They must equalize marginal products across managers with different z's. Since team production is constant returns to scale in managers' and workers' composite output, we can aggregate over teams (each led by a single manager) in (1) to write sectoral output as

$$Y_i = A_i \left[\eta_i^{\frac{1}{\omega}} X_{iz}^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} X_{ih}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{7a}$$

$$X_{iz} = M(z) K_{iz}^{\alpha} Z_i^{1-\alpha}, \qquad X_{ih} = \left[\int_j \nu_i(j)^{\frac{1}{\sigma}} T_i(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{7b}$$

where K_{iz} is the total amount of capital allocated to managers and $Z_i \equiv \int z l_{iz}(s) ds$. Similarly, the sectoral task composite X_{ih} combines sectoral task aggregates

$$T_i(j) = M(j) K_{ih}(j)^{\alpha} H_i(j)^{1-\alpha}, \tag{8}$$

where $K_{ih}(j)$ is the total amount of capital allocated to task j in sector i.

We characterize the planner's solution in three steps: (i) the optimal allocation of physical capital across tasks within a sector; (ii) the optimal worker assignment across tasks within a sector; and (iii) the optimal allocation of managers vs. workers within a sector. Section 3.3 combines these rules into a within-sector solution and expresses sectoral output in terms of assignment rules and sectoral aggregates. Section 3.4 establishes existence and uniqueness of relative service prices. We then derive equilibrium wages and comparative statics.

Capital allocation within sectors The planner equalizes the marginal product of capital across tasks. Given the technologies in (7)-(8), this means that all capital decisions can be expressed as a linear function of the capital used in task 0. Specifically,

$$\pi_{ih}(j) \equiv \frac{K_{ih}(j)}{K_{ih}(0)} = \left[\frac{\nu_i(j)}{\nu_i(0)} \right]^{\frac{1}{\sigma}} \cdot \left[\frac{T_i(j)}{T_i(0)} \right]^{\frac{\sigma-1}{\sigma}}, \tag{9}$$

with which we can express the worker task composite X_{ih} in (7) as

$$X_{ih} = \nu_i(0)^{\frac{1}{\sigma-1}} \Pi_{ih}^{\frac{\sigma}{\sigma-1}} T_i(0), \quad \text{where } \Pi_{ih} \equiv \int_j \pi_{ih}(j) dj. \tag{10}$$

Of course, marginal products must also be equalized across the managerial task and the rest. Using (10) we can define

$$\pi_{iz} \equiv \frac{K_{iz}}{K_{ih}(0)} = \left(\frac{\eta_i}{1-\eta_i}\right)^{\frac{1}{\omega}} \cdot \left(\frac{X_{iz}}{X_{ih}}\right)^{\frac{\omega-1}{\omega}} \cdot \Pi_{ih}, \tag{11}$$

which does not vary with j. Equations (9) and (11) subsume the capital allocation decisions into the labor allocation rules through $\pi_{ih}(j)$ and π_{iz}.

Sorting workers across tasks within sectors Under strict log-supermodularity and the regular interior characterization just stated, there is a continuous assignment function $\hat{j} : [0, h_M] \mapsto \mathcal{J}$ such that $\hat{j}(h)$ is strictly increasing, and $\hat{j}(0) = 0$, $\hat{j}(h_M) = J$.[18] That is, workers sort positively into tasks, with skill h assigned to job $\hat{j}(h)$. Since $\hat{j}(h)$ is strictly increasing, we can also define its inverse $\hat{h} : \mathcal{J} \mapsto [0, h_M]$.

The assignment functions $\hat{j}(h)$ and $\hat{h}(j)$ are identical across sectors and hence are not indexed by i. Otherwise, the planner would be able to reallocate h across sectors and increase output. So the planner's problem of choosing $l_{ih}(s, j)$ has two parts: One of choosing $l_h(s, j)$, the number of individuals with skill s assigned to task j *regardless of sector*, and the other of choosing $q_{ih}(j)$, the fraction of task j workers allocated to sector i, which satisfies $\sum_{i\in\{m,s\}} q_{ih}(j) = 1$. That is, we can write $l_{ih}(s, j) = q_{ih}(j) \cdot l_h(s, j)$.

Sorting managers vs. workers within a sector Standard Roy selection with two-dimensional heterogeneity implies a cutoff rule $\tilde{z}(h)$ such that for every h, individuals with z above $\tilde{z}(h)$ become managers and those below become workers; at equality either occupation is optimal. Since we know that workers sort positively into j, we also define $\hat{z}(j) = \tilde{z}(\hat{h}(j))$ for all $j \in \mathcal{J}$. As with workers, the manager selection rule must be identical across sectors.

Similar to Lucas (1978), $\hat{z}(j)$ is chosen so that the marginal product of the threshold manager is equalized between task z and j, so

$$\hat{z}(j) \big/ b(\hat{h}(j), j) = \pi_{ih}(j) Z_i \big/ \pi_{iz} H_i(j), \quad \forall j \in \mathcal{J} \tag{12}$$

and in particular, with some abuse of notation,

$$\hat{z} \equiv \hat{z}(0) = \tilde{z}(0) = Z_i \big/ \pi_{iz} H_i(0), \tag{13}$$

since we normalized $b(0, 0) = 1$. Equation (13) is the counterpart of (11): it normalizes the total productivity of managers in sector i by task-0 productivity $H_i(0)$. In the next subsection, we normalize all other workers' productivities by $H_i(0)$ as well.

3.3 Within-Sector Solution

First consider the rule $l_{iz}(s)$. The occupation-choice cutoff $\tilde{z}(h)$ determines the economy-wide pool of managers. For sectoral headcount identities, we use allocations with the same mean managerial skill $\bar{z}$ in both sectors. Equality of higher moments or entire skill distributions is unnecessary. For local comparative statics, equality of the baseline means and their first-order responses suffices. A convenient sufficient implementation gives each sector a constant fraction of every selected managerial type. Writing $l_z(s) = \sum_i l_{iz}(s)$, that implementation is

$$l_{iz}(s) = q_{iz} l_z(s), \qquad q_{iz} = Z_i/Z, \qquad \sum_{i\in\{m,s\}} q_{iz} = 1. \tag{14}$$

[18] For a more formal proof, refer to Lemma 1 in Costinot and Vogel (2010).

In this implementation, q_{iz} is independent of (h, z) within the selected pool; it need not be one half. More generally, $q_{iz} = Z_i/Z$, and common sectoral means alone imply the same aggregate headcount identity. In particular,

$$\int l_{iz}(s)ds = q_{iz} \int [1 - F(\tilde{z}(h) \mid h)]dG(h), \tag{15}$$

with

$$Z = \int \int_{\tilde{z}(h)}^{\infty} z \, dF(z \mid h)dG(h), \qquad Z_i = q_{iz}Z.$$

The planner requires the sectoral totals Z_i but is indifferent among assignments of selected managers delivering those totals. The constant-fraction convention selects an allocation and makes sectoral managerial headcounts proportional to effective managerial services. It applies separately in each equilibrium. It does not fix who becomes a manager: changes in $\tilde{z}(h)$ alter the selected pool before it is divided between sectors.

Next consider the planner's choice of $l_h(s, j)$. The characterization is similar to Costinot and Vogel (2010) and summarized in Lemma 1.

Lemma 1 *Define*

$$B_j(j; \hat{h}) \equiv \exp\left[\int_0^j \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj'\right]. \tag{16}$$

At the optimum, the productivity of all workers assigned to task j can be expressed as

$$H(j) = b(\hat{h}(j), j) \cdot F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j)) \cdot \hat{h}'(j), \quad \forall j \in \mathcal{J}, \tag{17}$$

and in each sector i,

$$H_i(j) = q_{ih}(j)H(j) = H_i(0) \cdot \pi_{ih}(j)B_j(j; \hat{h}), \tag{18}$$

Equation (17) simply states that total worker productivity in task j is the product of the infinitesimal mass of individuals assigned to task j, times their effective productivity. Equation (18) is the counterpart of (9): the marginal products of labor are equated at every point along $\mathcal{J}$. Similarly, all manager allocations can also be normalized by $\hat{z}$.

Corollary 1 *Define the counterpart of B_j in* (16)*:*

$$B_h(h; \hat{j}) \equiv \exp\left[\int_0^h \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh'\right]. \tag{19}$$

At the optimum, the managerial cutoff rules $\hat{z}(j)$ and $\tilde{z}(h)$ can be expressed as

$$\hat{z}(j) = \hat{z} \cdot b(\hat{h}(j), j) \big/ B_j(j; \hat{h}) = \hat{z} \cdot B_h(\hat{h}(j); \hat{j}) \quad \Leftrightarrow \quad \tilde{z}(h) = \hat{z} \cdot B_h(h; \hat{j}). \tag{20}$$

The corollary makes all manager cutoff rules except $\hat{z}$ (the threshold at $h = 0$) redundant, since they can be expressed only in terms of $\hat{z}$ in (13) and $\hat{h}(j)$. In what follows, we suppress the dependence of B_j and B_h on $\hat{h}$ and $\hat{j}$ unless necessary.

Let L_i denote total employment in sector i, including managers. We can now show that the within-sector solution is characterized only by $\{\hat{h}(j), \hat{z}\}$:

Proposition 1 *Suppose $[q_{ih}(j)]_{j=0}^{J}$ and q_{iz} are given. Under the regular interior characterization above, the within-sector equilibrium characterization depends only on $\left\{[\hat{h}(j)]_{j=0}^{J}, \hat{z}\right\}$, and does not depend on sectoral aggregates K_i and L_i. Proposition 2 in the next subsection establishes existence with endogenous sectoral weights.*

A within-sector solution yields a simple expression for sectoral output:

Corollary 2 *At a planner's optimum, sectoral output can be expressed as*

$$Y_i = \Phi_i \cdot K_i^{\alpha} L_i^{1-\alpha}, \quad \text{where } \Phi_i \equiv M(0) \cdot \psi_i \cdot \Pi_{ih}^{\frac{\sigma-\omega}{(\sigma-1)(1-\omega)}} \Pi_{K_i}^{\frac{\omega}{\omega-1}-\alpha} \Pi_{L_i}^{\alpha-1} \tag{21}$$

is the sectoral TFP, endogenously determined by the optimal allocation rules. Sectoral TFP can be decomposed into 3 parts:

1. *$M(0)$, which is common across both sectors and exogenous;*
2. *$\psi_i \equiv A_i(1-\eta_i)^{\frac{1}{\omega-1}} \nu_i(0)^{\frac{1}{\sigma-1}}$, which is also exogenous but sector-specific;*
3. *the part determined by $(\Pi_{ih}, \Pi_{K_i}, \Pi_{L_i})$, which is sector-specific and endogenously determined by the allocation rules $\hat{h}(j)$ and $\hat{z}$, where*

$$\Pi_{K_i} \equiv \Pi_{ih} + \pi_{iz} = K_i/K_{ih}(0) \text{ and } \Pi_{L_i} \equiv \Pi_{il} + (\hat{z}/\bar{z})\pi_{iz} = L_i/H_i(0) \tag{22}$$

are sectoral capital and employment in units of task-0 capital and effective worker skill, respectively; $\Pi_{il} \equiv \int \left[\pi_{ih}(j)/B_h(\hat{h}(j))\right] dj$, and $\bar{z} \equiv Z/\int [1 - F(\bar{z}(h)|h)]dG(h)$ is the mean of managers' productivity.

Sectoral TFP depends on occupational composition. Let R be the rental rate of capital, p_i the price of sectoral output, W_i the sectoral labor wage bill, and $\bar{w}_i = W_i/L_i$ sectoral average earnings. Common capital shares imply $RK_i = \alpha p_i Y_i$ and $W_i = (1-\alpha)p_i Y_i$, hence

$$\kappa \equiv \frac{L_s}{L_m} = \frac{K_s}{K_m}\frac{\bar{w}_m}{\bar{w}_s} = \frac{\gamma_s}{\gamma_m}\left(\frac{\Phi_s}{\Phi_m}\right)^{\epsilon-1}\left(\frac{\bar{w}_s}{\bar{w}_m}\right)^{-[\alpha+\epsilon(1-\alpha)]}. \tag{23}$$

Capital and wage-bill ratios coincide with the sectoral revenue ratio; headcount ratios also depend on relative average earnings. Thus relative TFP alone does not determine employment shares. Sectoral managerial headcounts use the common-mean allocation convention in (22).

Lemma 2 *Under Assumptions 2 and 3, the one-sector equilibrium has unique service prices. Under Assumption 1, atomless selected worker skills, and zero probability on the worker–manager indifference curve, occupational choices are also unique almost everywhere. This statement concerns equilibrium prices, not arbitrary imposed sectoral allocation weights.*

3.4 Existence and Uniqueness of Equilibrium

The equilibrium conditions above can be collected into a price problem. This separates the assumptions needed for equilibrium from the stronger restrictions used for particular comparative statics. Let $p_h(j)$ be the price of a unit of effective worker service in task j, and let p_z be the corresponding managerial price. These prices are common across sectors, with $p_z = w_z$ and $w_h(h) = \max_j p_h(j)b(h,j)$. The $q_{ih}(j)$ and q_{iz} remain sectoral allocation weights. Define

$$\mathcal{G}(p_h, p_z) = \int \max\left\{\max_{j\in\mathcal{J}} p_h(j)b(h,j),\ p_z z\right\} d\mu(h,z). \tag{24}$$

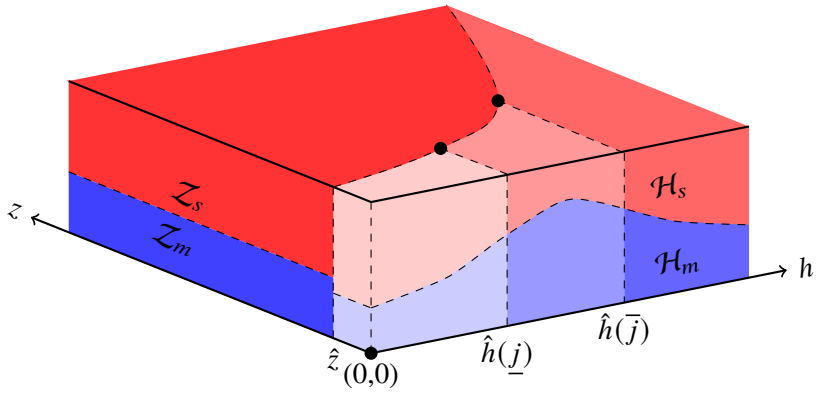


Fig. 6: Equilibrium

This is aggregate labor earnings at occupational choices induced by the prices. It is convex and homogeneous of degree one in prices.

The common capital share allows the rental rate to be factored out of the nested unit cost. Write $\Sigma = \alpha + (1-\alpha)\sigma$, $\Omega = \alpha + (1-\alpha)\omega$, and $\Xi = \alpha + (1-\alpha)\epsilon$. The remaining service-price index is

$$\begin{aligned} C_{ih}(p_h) &= \left[\int \nu_i(j) M(j)^{\sigma-1} p_h(j)^{1-\Sigma} dj\right]^{1/(1-\Sigma)}, \\ C_i(p_h, p_z) &= \left[\eta_i M(z)^{\omega-1} p_z^{1-\Omega} + (1-\eta_i) C_{ih}^{1-\Omega}\right]^{1/(1-\Omega)}, \\ C(p_h, p_z) &= \left[\sum_i \gamma_i A_i^{\epsilon-1} C_i^{1-\Xi}\right]^{1/(1-\Xi)}. \end{aligned} \tag{25}$$

Expressions with a unit elasticity are interpreted by their Cobb–Douglas limits. The full output unit cost is $\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} R^{\alpha} C^{1-\alpha}$. For finite positive elasticities, C is concave and strictly concave between nonproportional positive price vectors. Relative equilibrium prices solve

$$\max_{p_h \geq 0,\ p_z \geq 0} C(p_h, p_z) \quad \text{subject to} \quad \mathcal{G}(p_h, p_z) \leq 1. \tag{26}$$

The normalization fixes the common price scale; it imposes no restriction on the equilibrium allocation.

Proposition 2 (Existence and Uniqueness) *Suppose Assumptions 2 and 3 hold. Then a competitive equilibrium exists with unique relative service prices. Sectoral output and aggregate capital and effective-service allocations are also unique, and the final-good numeraire fixes absolute prices. Under Assumption 1, atomless selected worker skills, and zero probability on the worker–manager indifference curve, occupational choices and worker assignments to tasks are unique up to sets of individuals of measure zero. Individuals may remain indifferent between assignments to sectors or teams that deliver the same aggregate services.*

The proof is in Appendix C.4. Existence uses the technology conditions and finite skill moments; strict log-supermodularity and the stated selection conditions give unique occupational choices. The distributional and shape restrictions introduced below apply only to the corresponding comparative statics.

For illustrative purposes, the equilibrium skill allocation with a uniform $\mu(z, h)$ is depicted in Figure 6. Individuals in $\mathcal{Z}$ are managers, and those in $\mathcal{H}$ workers, while the subscripts s and m denote services and manufacturing. Workers sort into tasks indexed by j according to $\hat{h}(j)$. The different masses of sectoral

employment across tasks are due to the task intensity parameters $\nu_i(j)$ and η_i.

3.5 Equilibrium Wages and Prices

The solution $\hat{h}(j)$ and $\hat{z}$ give all the information needed to derive equilibrium prices (which are unique). The price of the final good can be normalized to 1:

$$P = 1 = \left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \qquad p_i = [Y_i/\gamma_i Y]^{-\frac{1}{\epsilon}} \tag{27}$$

and sectoral output prices p_i are obtained by plugging in the sectoral production function (21). Let R denote the rental rate of capital and $w_h(h)$ the wage of a worker with skill h. Equal task-service prices imply that $w_h(0)$ can be found from either sector:

$$w_h(0) = \frac{1-\alpha}{\alpha} \cdot \frac{RK_{ih}(0)}{H_i(0)} \cdot b(0,0) = \frac{1-\alpha}{\alpha} \cdot \frac{\Pi_{L_i}}{\Pi_{K_i}} \cdot R\frac{K_i}{L_i},$$

where the second equality follows from (22) and the normalization $b(0,0) = 1$. Similarly, all workers earn their marginal product, so we can write

$$w_h(h) = w_h(0) \cdot B_h(h). \tag{28}$$

To sign the slope of worker earnings and the selection curve, assume

Assumption 4 *Along the baseline assignment,* $\partial_h \log b(h, \hat{j}(h)) \geq 0$.

This assumption says that increasing a worker's skill does not reduce productivity at the task actually assigned in equilibrium. No sign is imposed at unassigned skill–task combinations, and since

$$\frac{d \log B_h(h; \hat{j})}{dh} = \frac{\partial \log b(h, \hat{j}(h))}{\partial h}, \tag{29}$$

Assumption 4 implies that $w_h(h)$ is weakly increasing in h. By (20), the threshold $\tilde{z}(h) = \hat{z} \cdot B_h(h)$ is also weakly increasing. At each $h \in \mathcal{H}$, individuals with managerial skill $z = \tilde{z}(h)$ are indifferent between management and worker employment, so the price per unit of managerial skill, w_z, satisfies

$$w_z \tilde{z}(h) = w_h(h) \quad \Rightarrow \quad w_z = w_h(0)/\hat{z}, \tag{30}$$

and (20) and (28) guarantee that the first equality holds for all h.

Productivity, requirements, and relative wages. We now consider how task productivity, requirements, and skill supply affect employment and wages. For each of the three occupational experiments, we distinguish two wage comparisons: earnings differences among workers and earnings differences between workers and managers. These comparisons hold individual skills fixed; reassignment and selection connect them to occupational employment and mean earnings. Throughout these comparisons, write $t(h) = \tilde{z}(h) = w_h(h)/w_z$ for the managerial selection cutoff and $w(h) = w_h(h)$ for worker earnings. We begin with automation of middle-skill tasks, then turn to skill requirements and relative managerial scarcity, before considering sectoral reallocation. Each proposition isolates one force under explicit sufficient conditions. The historical calibration allows these forces to change jointly and does not impose the comparative-static restrictions.

The skill-supply and requirement comparisons below concern a one-sector equilibrium. They use finite positive ω, σ and a regular, differentiable interior assignment with $\partial_{hj} \log b > 0$. Conditional

densities and the selected-supply derivatives used in Appendix D.2 are smooth and finite near the baseline selection curve. Strict managerial employment changes additionally require a positive selection density on a set of worker skills with positive measure. These local regularity conditions are additional to the general equilibrium assumptions; they impose no ordering of ω and σ and do not require vanishing log-supermodularity. The managerial relative-gain result uses $\omega < 1$; the requirement wage-gradient result does not, while its managerial wage-bill conclusion is stated separately for that case. Weak inequalities refer to first-order responses; a zero derivative alone does not sign higher-order finite changes.

3.6 Task Automation and Reallocation

Given the equilibrium allocation, we can extend Costinot and Vogel (2010) to conduct several comparative statics, such as the effect of a change in task-specific TFPs $M(j)$.[19] In particular, we are interested in the effect of an increase in the TFP of middle-skill tasks. As shown in Section 5, these tasks are "routine jobs," and hence the productivity growth specific to them is called "routinization."

For the local task-automation result, we impose the following sufficient conditions.

Assumption 5 (Regular weak comparative advantage) *On the compact worker and task domains, consider*

$$b_\delta(h, j) = b_0(h)k_0(j)\exp\{\delta u(h, j)\}, \qquad \delta > 0,$$

where b_0, k_0, u are C^3, $b_0, k_0 > 0$, and $u_{hj} > 0$ along the reference assignment, defined in Appendix D.1, equation (49)*. Strict log-supermodularity holds for the baseline family, but the cross-partial need not be bounded away from zero at unassigned skill–task combinations. All other primitives are fixed. The worker density g and the baseline unit-cost task weight $\nu(j)M(j)^{\sigma-1}$ are smooth and strictly positive. Conditional worker participation is smooth and positive near the equilibrium cutoffs, with smooth conditional densities there and a finite total managerial first moment. No smoothness restriction is imposed on the conditional distribution away from these cutoffs. Use the regular interior characterization for sufficiently small δ.*

The family specifies how log-supermodularity becomes weak while holding preferences, skill distributions and task productivities fixed. Its smoothness is used for the local response and continuation argument, not for the general equilibrium theorem.

Assumption 6 (Participation ordering at the reference selection curve) *At the separable reference equilibrium $\delta = 0$, normalize $p_z = 1$ and write $t_0(h) = q_0 b_0(h)$, where q_0 is the equilibrium price per unit of the separable worker input $b_0(h)$. Define*

$$e_0(h) = \frac{t_0(h)f(t_0(h) \mid h)}{F(t_0(h) \mid h)}, \qquad \bar{e}_0 = \frac{\int t_0(h)F(t_0(h) \mid h)e_0(h)dG(h)}{\int t_0(h)F(t_0(h) \mid h)dG(h)}.$$

The elasticity $e_0(h)$ is nondecreasing, and the limiting managerial wage-bill share, denoted τ_{z0}, satisfies

$$(1-\omega)\tau_{z0}[\bar{e}_0 - e_0(0)] < (1-\sigma)[\bar{e}_0 + \Omega\tau_{z0}], \qquad \Omega = \alpha + (1-\alpha)\omega. \tag{31}$$

These restrictions apply only along the reference selection curve. Equivalently, for $\mathcal{F}(x, h) = \log F(e^x \mid h)$, the ordering is $\mathcal{F}_{xh} + \mathcal{F}_{xx}\partial_h \log b_0 \geq 0$ there. It does not require a global ordering of conditional skill

[19] A change in factor-neutral task-TFP, $M(j)$, is different from an increase in the amount of skill working in any given task. Since we model worker skill as human capital, the qualitative effect of a change in task-TFP is the same as if it were only capital-augmenting—e.g., a fall in the price of task-specific capital.

distributions. A simpler, more restrictive sufficient check is

$$\tau_{z0} \leq \frac{1-\sigma}{1-\omega}. \tag{32}$$

The bounds apply to the managerial share of labor earnings.

Polarization Consider the comparative statics for $\hat{h}(j)$ and $\hat{z}$ within a sector i, ignoring sectoral reallocation. Suppose that there is an increase in the exogenous productivity, $M(j)$, of middle-skill j-tasks.[20]

Proposition 3 (Routinization and Polarization) *Consider a one-sector equilibrium and* $0 < \underline{j} < \overline{j} < J$. *Suppose task productivity rises uniformly on* $\mathcal{J}_1 = (\underline{j}, \overline{j})$*:*

$$M_1(j) = M(j)\exp\{m\mathbf{1}_{\mathcal{J}_1}(j)\}, \qquad m > 0,$$

with all other primitives held fixed and jumps permitted at the interval boundaries. If $0 < \omega < 1$ *and* $0 < \sigma < 1$*, with no ordering between them, then under Assumptions 1, 2, 3, 5 and 6 along the baseline family, for every sufficiently small positive* δ *and then every sufficiently small positive* m*:*

1. *There is exactly one* $j^* \in \mathcal{J}_1$ *such that* $\hat{h}_1(j) > \hat{h}(j)$ *for* $0 < j < j^*$ *and* $\hat{h}_1(j) < \hat{h}(j)$ *for* $j^* < j < J$.
2. *The managerial cutoff falls at every fixed worker skill* h*:* $t_1(h) < t(h)$*. Thus every fixed managerial type gains relative to every fixed worker type. In particular* $\hat{z}_1 < \hat{z}$ *and* $w_{z1}/w_{h1}(0) > w_z/w_h(0)$.

Part 1 describes polarized task assignment among workers: below the crossing, fixed worker types move toward less skill-intensive tasks; above it they move toward more skill-intensive tasks. It also gives polarized worker wage growth. If h^* is the worker type whose assignment is unchanged, then

$$\log\frac{w_1(h)}{w_1(h^*)} - \log\frac{w(h)}{w(h^*)} = \int_{h^*}^{h}\left[\partial_h \log b(h, \hat{j}_1(h)) - \partial_h \log b(h, \hat{j}(h))\right] dh > 0 \quad (h \neq h^*).$$

Part 2 isolates a complementary managerial wage channel. Task automation raises managerial earnings relative to each fixed worker type, and, when the selection margin is active, management entry changes occupational composition. The effect on mean managerial earnings also depends on entry, as we show below in Section 3.8, equation (36). Likewise, the assignment crossing alone does not sign occupational headcounts or capital shares when workers can enter management. Figure 7 illustrates the within-sector assignment comparison.

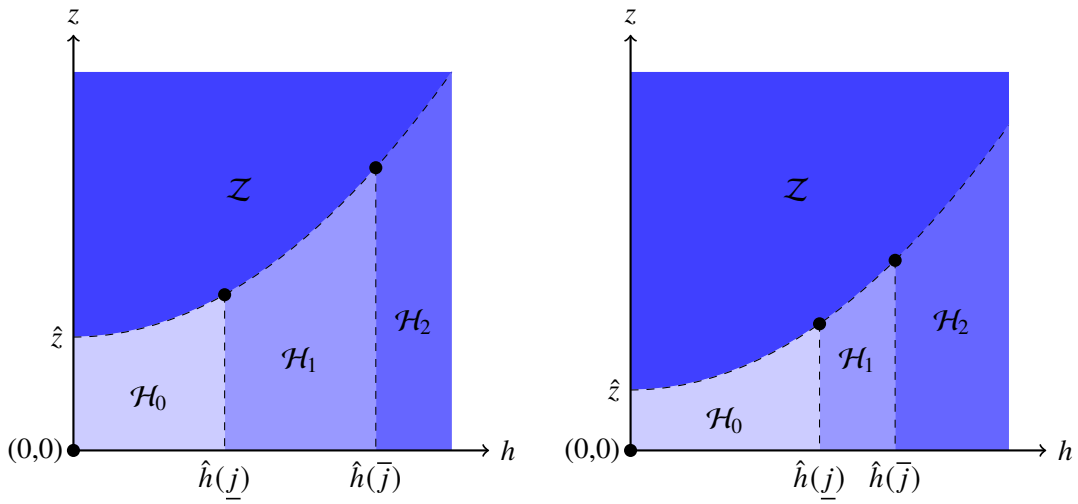
Complementarity across worker tasks generates reallocation away from the improved middle interval, as in Costinot and Vogel (2010), while endogenous managerial selection can oppose that force. The share restriction ensures that selection does not overturn the polarized assignment in this local experiment. The quantitative exercise below evaluates finite changes and the resulting occupational composition.

3.7 Upskilling and Worker Wage Growth

We next change the requirement function while holding task productivities fixed. For the analytical comparison, let

$$b(h, j) = h - \chi(j), \qquad \chi' > 0, \quad \max_j \chi(j) < 0, \quad \chi(0) = -1.$$

[20] This *within-sector* exercise is similar to Lemma 6 in Costinot and Vogel (2010), except that we have capital and, more importantly, the second dimension of skill that qualitatively separates managers from workers.



(a) Equilibrium with low $M(j),\ j \in \mathcal{J}^1$

(b) Equilibrium with high $M(j),\ j \in \mathcal{J}^1$

Fig. 7: Comparative Static, Within-Sector.

Write the change in the requirement profile as

$$\chi_\varepsilon(j) = \chi(j) + \varepsilon d(j), \qquad A(j) = \frac{d(j)}{\chi'(j)}.$$

Here $d(j)$ specifies how requirements change across tasks. The function $A(j)$ expresses the change in units of the task index: to first order, $\chi_\varepsilon(j) = \chi(j + \varepsilon A(j))$. A proportional index stretch corresponds to $A(j) = j$; a parallel shift or a change concentrated in high-skill tasks gives a different experiment. We restrict ε so that $b_\varepsilon > 0$ and $\chi'_\varepsilon > 0$. The normalization $\chi(0) = -1$ gives $b(0,0) = 1$ at the baseline. If $d(0) > 0$, perturbed bottom productivity is $1 - \varepsilon d(0)$, and occupational comparisons use the actual earnings ratio $w_h(h)/w_z$.

At a fixed task, the return to worker skill changes by $d(j)/b(h,j)^2$. Including reassignment, the exact local worker wage-gradient response is

$$\partial_\varepsilon \partial_h \log w(h) = \frac{d(j) + \chi'(j)\partial_\varepsilon \hat{j}(h)}{b(h,j)^2}, \qquad j = \hat{j}(h). \tag{33}$$

Thus the sign of $d(j)$ alone, or a steeper requirement profile by itself, does not establish the equilibrium wage ranking. The following proposition gives a sufficient class of profile changes with an upskilling interpretation: wage growth tilts toward higher-skill workers even after reassignment. This is a partial effect to be combined with the relative-productivity change.

Assumption 7 (Selection elasticity for requirement changes) *Along the regular baseline selection curve, worker participation has constant elasticity*

$$\frac{t(h) f(t(h) \mid h)}{F(t(h) \mid h)} = k \geq 0.$$

Only the elasticity at the selected cutoffs enters the local wage-gradient proof. No common scale family, global power law, or upper-tail shape is required. A primitive sufficient example is $z = S(h)U$, with U independent of h, finite mean, and $F_U(u) = \rho u^k$ on a lower body containing the selected arguments,

where $k > 0$ and $0 < \rho \leq 1$. Outside that region the distribution can be any finite-mean continuation. Appendix D.3 gives a primitive interior-selection bound and a smooth common-support example. These are illustrations rather than additional model assumptions.

Proposition 4 (Skill Requirements and Wage Growth) *Consider a one-sector equilibrium satisfying Assumption 7 and the local regularity conditions stated above, with any finite positive ω, σ. Let $A_0 = \log \nu + (\sigma - 1) \log M$ and A be twice continuously differentiable. Suppose requirements change smoothly to $\chi_\varepsilon = \chi + \varepsilon d$, with task productivities and the skill distribution held fixed, where*

$$d(j) = \chi'(j)A(j), \qquad A(0), A(J) \geq 0, \qquad A \not\equiv 0, \qquad -[A' + AA_0']' \geq 0.$$

Then:

1. *Let $y(h) = \partial_\varepsilon \log[w_h(h)/w_z]$ denote relative wage growth at a fixed worker skill. The worker-wage-bill-weighted average of y has the sign of $1 - \omega$. If $0 < \omega < 1$, workers in a nonempty upper interval have $y(h) > 0$ and gain relative to every fixed managerial type, while the managerial share of the labor wage bill falls. In that case, lower-skill workers may gain or lose relative to managers; if any lose, they lie below a unique zero of y. At $\omega = 1$ the managerial wage-bill share is unchanged, and for $\omega > 1$ it rises. The effect on managerial headcount is ambiguous.*
2. *For every $h_2 > h_1$,*

$$\partial_\varepsilon \log \frac{w_h(h_2)}{w_h(h_1)} > 0.$$

Thus wage growth is strictly increasing in worker skill, including the equilibrium reassignment response.

The joint restriction is $-A'' - A'A_0' - AA_0'' \geq 0$; it concerns the direction of the requirement change together with baseline composite task weights. It implies $A > 0$ in the interior under the stated endpoint and nontriviality conditions. The separate restrictions $A \geq 0$, $A' \geq 0$, $A'' \leq 0$, $A_0' \leq 0$, and $A_0'' \leq 0$ are an easy sufficient example, but are not each necessary. The index stretch $A = j$ is one example under those weight restrictions. This result does not assert that d itself is increasing or that every upward requirement change has the same wage effect.

Among workers, this class of requirement changes raises wage growth with skill, including the response of occupational assignment. Between workers and managers, sufficiently high-skill workers gain relative to every fixed managerial type when $\omega < 1$, while lower-skill workers may gain or lose. The second comparison can work against the managerial gains from relative scarcity, which we study next in Proposition 5. Neither comparison alone determines the change in occupational mean wages, since the composition of workers and managers also changes. The quantitative exercise below combines these forces and accounts for selection.

The wage ranking does not itself sign reassignment. Writing $y_J(j) = y(\hat{h}(j))$ on the baseline assignment,

$$\partial_\varepsilon \hat{j}_\varepsilon(h) = \frac{y_J'(j)}{\partial_{hj} \log b(h, j)\, \hat{h}'(j)} - A(j), \qquad j = \hat{j}(h). \tag{34}$$

For $A = j$, Appendix D.3 gives an additional sufficient restriction for upward sorting. This allocation result is distinct from either wage comparison.

Appendix D.3 also constructs a smooth common-support skill distribution under which the requirement and managerial-sorting results both apply. This links the examples across the two comparisons without

adding restrictions to the general equilibrium model.

3.8 Relative Productivity, Skills, and Wages

We now distinguish managerial earnings from the earnings of other high-skill workers. Relative productivity and managerial skill supply can change the scarcity value of management even when task requirements are held fixed.

Write worker-task productivity as $M(j) = M_h \bar{M}(j)$, with a fixed task profile $\bar{M}(j)$. To study a productivity change that raises managerial earnings relative to workers, define

$$a = \log \frac{M_h}{M(z)}.$$

A rise in a means that managerial productivity grows less than the common worker-productivity component, not necessarily that managerial productivity falls in levels. Because the two inputs are complementary, this relative productivity lag raises the scarcity value of managerial services. Corollary 3 gives the reverse comparison.

The same experiment has a skill-supply interpretation. Write $z = s_z z^0$ for a uniform scaling of each person's managerial skill, with the joint distribution of (z^0, h) fixed. Then managerial technology depends on $M(z)s_z^{1-\alpha}$, and the effective relative change is

$$da = d \log M_h - d \log M(z) - (1 - \alpha) d \log s_z. \tag{35}$$

If μ_z denotes population mean managerial skill, $s_z = \mu_z / \mu_z^0$ under this scale change. A lower μ_z therefore has the same relative effect as lower $M(z)$, with factor $1 - \alpha$. An arbitrary mean change that alters dispersion or dependence on h is not covered. Comparisons follow the same individuals and express selection cutoffs in original z^0 units. Uniform worker productivity is also distinct from changing the skill argument inside $b(h, j)$.

Assumption 8 (Participation elasticity for managerial sorting) *Along the baseline selection curve $t(h) = w_h(h)/w_z$, the elasticity*

$$e(h) = \frac{t(h) f(t(h) \mid h)}{F(t(h) \mid h)}$$

is nondecreasing in worker skill h.

This restriction concerns participation at the selected cutoffs, not the whole joint skill distribution. A primitive sufficient example is $z = S(h)U$, with U independent of h, finite mean, and nonincreasing lower-tail elasticity $r_U(u) = u f_U(u)/F_U(u)$ over the selected range. It suffices that $S'(h)/S(h) \geq \partial_h \log b(h, \hat{j}(h))$ along the baseline assignment. Restrictions away from that curve are unnecessary. Positive correlation alone would not establish the required elasticity ordering.

Proposition 5 (Relative Managerial Scarcity) *Consider the one-sector equilibrium described above, with $0 < \omega < 1$ and any finite $\sigma > 0$. Suppose $a = \log[M_h/M(z)]$ increases by a small amount, while the skill distribution, b, and $\bar{M}(j)$ remain unchanged. Then:*

1. *For every fixed managerial skill $z > 0$ and worker skill h,*

$$\partial_a \log \frac{w_z z}{w_h(h)} > 0.$$

The entire selection curve $\tilde{z}(h)$ falls, and the managerial share of the labor wage bill rises. Managerial employment weakly rises; its first-order response is strictly positive when the selection density is positive on a set of worker skills with positive measure.

2. *Under Assumption 8, for any $h_2 > h_1$,*

$$\partial_a \log \frac{w_h(h_2)}{w_h(h_1)} \geq 0.$$

Workers move weakly toward more skill-intensive tasks: $\partial_a \hat{j}(h) \geq 0$ and $\partial_a \hat{h}(j) \leq 0$. If participation elasticity is nonconstant, wage differences strictly widen and interior task responses are strict. If it is constant, worker wage growth is identical across skills and the within-worker inequality effect is zero.

Uniform changes in managerial-skill scale have these same effects for the effective change in (35)*, with original skill units used for the selection curve.*

Corollary 3 (The reverse relative-productivity change) *A rise in $M(z)/M_h$, or a uniform rise in managerial skill $z = s_z z^0$ holding productivity fixed, reverses the signs in Proposition 5. Managers lose relative to every fixed worker type; the managerial wage-bill share falls, and managerial employment weakly falls, with a strict first-order fall under the active-selection condition. Under Assumption 8, worker wage differences weakly compress and workers move weakly toward less skill-intensive tasks.*

Among workers, managerial scarcity weakly widens wage gaps under the participation-elasticity ordering: selection into management also changes the allocation of worker skills across tasks. Between workers and managers, every fixed managerial type gains relative to every worker type, and managerial employment weakly rises. The constant-elasticity case separates the two comparisons: managerial relative earnings rise while wage gaps among workers remain unchanged. Occupational mean earnings also depend on the entrants and leavers, as the following identity makes explicit.

Partial effects and quantitative attribution. The quantitative exercise allows productivity, requirements, and skill supply to change jointly. It measures their interaction over finite changes and accounts for selection when computing occupational mean earnings.

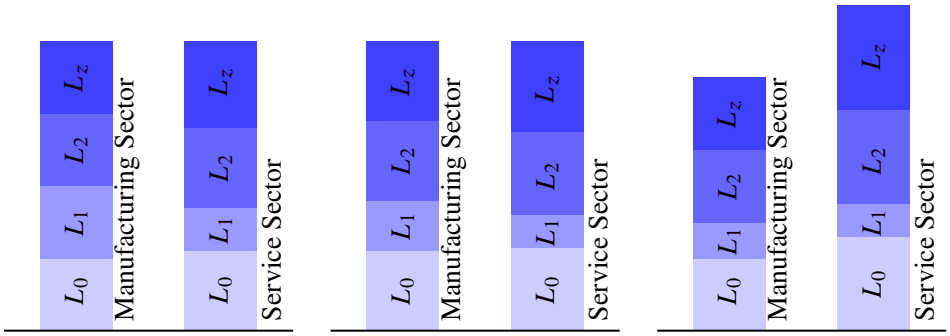
Occupational mean earnings. Let L_z denote managerial employment and τ_z its share of the labor wage bill. For each experiment,

$$\frac{\bar{w}_z}{\bar{w}_h} = \frac{\tau_z}{1-\tau_z}\frac{1-L_z}{L_z}, \qquad \partial_a \log \frac{\bar{w}_z}{\bar{w}_h} = \frac{\partial_a \tau_z}{\tau_z(1-\tau_z)} - \frac{\partial_a L_z}{L_z(1-L_z)}. \tag{36}$$

Here $\bar{w}_z$ and $\bar{w}_h$ are mean managerial and worker earnings. The managerial mean premium therefore rises when the wage-bill term in (36) exceeds the employment-composition term. Changes in composition also enter the mean wage of high-skill worker occupations. Appendix D.3 gives an illustrative mean-earnings comparison under a bounded-power skill distribution.

3.9 Sectoral Reallocation

Routinization can generate structural change through differential exposure to a common task-productivity shock. The sector using routine tasks more intensively experiences a larger productivity gain. Its relative output price falls, and complementarity between sectoral outputs reduces its expenditure share. Employment also depends on the occupational composition of the sector's wage bill. Proposition 6 gives



(a) Low $M(j)$, $j \in \mathcal{J}^1$ (b) High $M(j)$, $j \in \mathcal{J}^1$, κ fixed (c) High $M(j)$, $j \in \mathcal{J}^1$

Fig. 8: Comparative Statics, Across Sectors.

alternative local conditions under which the more exposed sector loses employment share. They concern headcount composition as well as relative sectoral revenue.

For this proposition, initially $M(j) = M_0$ for all worker tasks and

$$M_a(j) = M_0 \exp\{a\mathbf{1}_{\mathcal{R}}(j)\}, \qquad r_i = \int_{\mathcal{R}} \nu_i(j)dj, \qquad r_m > r_s,$$

where $\mathcal{R} \subset (0, J)$ is the routine interval. Sectoral productivities and all other primitives are fixed. To isolate sectoral exposure, first use the homogeneous-worker benchmark $b(h, j) = 1$. Normalize the managerial service price to one and denote the common worker wage and managerial selection cutoff by t. Managerial skill has finite mean and a smooth density near this interior cutoff; the density may be zero, and atoms away from the cutoff are allowed. For these local headcount comparisons, sectoral managerial means and their first-order responses are equal, so managerial employment equals effective services divided by the common mean $\bar{z}$ to the required order. Sectoral distributions can otherwise differ. The constant-fraction allocation in (14) is one sufficient implementation. General equilibrium alone does not distinguish other assignments of managers that supply the same effective services.

Proposition 6 (Routinization and Structural Change) *Suppose $0 < \alpha < 1$ and $\omega, \sigma > 0$ are finite. If either set of conditions below holds, then a small increase in a raises Φ_m/Φ_s and Y_m/Y_s, and lowers p_m/p_s, K_m/K_s, and L_m/L_s. Hence manufacturing's employment share falls.*

1. ***Equal managerial weights.*** *The initial weights satisfy $\eta_m = \eta_s$, $0 < \epsilon < 1$, and*

$$(1-\epsilon)\theta + (1-\omega)\frac{(1-\phi)\theta(1-\theta)}{\phi + (1-\phi)\theta} > 0, \qquad \phi = t/\bar{z},$$

   where θ is the common baseline worker wage-bill share. This condition is automatic when $0 < \omega \leq 1$; it also permits $\omega > 1$ when relative-revenue contraction dominates headcount composition.

2. ***Unequal managerial weights.*** *The initial weights need not be ordered, and $0 < \epsilon < 1$. The exact baseline price and headcount conditions in Appendix D.5, equations (77) and (76), hold. No parametric managerial-skill distribution is required. A primitive sufficient example has $\eta_m \leq \eta_s$, $0 < \omega < 1$, a selected Pareto upper tail with exponent $q > 1$, and*

$$\epsilon \leq 1 - \frac{1-\omega}{q}.$$

The tail need hold only above some $z_0 < t$; the distribution below z_0 is unrestricted.

For each sufficiently small fixed positive shock, the strict comparisons persist under sufficiently small smooth perturbations of the respective benchmark with continuous equilibrium prices, aggregate effective allocations, and sectoral managerial means. In particular, part 1 permits a small positive gap $\eta_s - \eta_m$. Both parts permit sufficiently small positive worker heterogeneity $b_\delta(h, j) = \exp\{\delta\psi(h, j)\}$, where $\psi(0, 0) = 0$ and $\psi_{hj} > 0$, with ψ smooth, bounded, and uniformly continuous in the task index uniformly over worker skill. Extending the signs of the local derivatives themselves additionally requires differentiable continuations of prices, capital and headcount allocations, and sectoral managerial means whose derivatives are continuous at the benchmark.

The two parts give alternative sufficient conditions for structural change. The homogeneous-worker benchmark isolates sectoral exposure; positive δ restores strict log-supermodularity. This extension requires small worker productivity dispersion, not only a small cross-partial. The Pareto tail provides one example for part 2. The quantitative calibration is not restricted to either regime.

The proof in Appendix D.5 separates the expenditure response from the change in headcounts per unit of wage bill. The exact headcount condition allows the relative-revenue contraction to dominate an opposing composition response. The simpler complementary-manager configuration reinforces the decline in part 1; part 2 permits larger managerial-intensity differences. Relative real output grows even as employment share falls. All of these changes occur in the same equilibrium comparison.

Structural change also contributes to aggregate occupational change through composition. If manufacturing has the higher routine employment share, its contraction reduces the aggregate routine share; if services has the higher managerial employment share, the same reallocation raises the aggregate managerial share. Appendix D, Lemma 3, states this accounting decomposition. The decomposition evaluates within-sector and between-sector changes at the full equilibrium. Figure 8 illustrates the within-sector and between-sector components of this adjustment.

4 Calibration

We first calibrate the model to 1980, then match 2019 by changing task-specific productivities, the Pareto skill-distribution parameters, and the skill requirements χ_j. Task-productivity growth may differ across all ten worker occupations and management. The distributional changes alter the shape of worker and managerial skills, as specified below. Sectoral task weights, sector-specific productivities, and substitution elasticities are common to both dates. Capital per worker is taken from the data at each date.

We estimate the aggregate technology (4) separately from sectoral prices and output. Within the model, the 1980 calibration matches occupational employment shares and relative wages, the manufacturing employment share, and output per worker. The 2019 calibration uses the same relative-wage targets, together with employment shares within sectors and aggregate output per worker. The elasticities σ and ω are disciplined jointly with task-productivity growth by the change between the two dates, but do not themselves change over time. The intervening decades (1990, 2000 and 2010) are computed by interpolating the changing parameters between the calibrated endpoints.

4.1 Parametrization

Discrete log-supermodularity In the quantitative analysis, we collapse the continuum of horizontally differentiated worker tasks into 10 groups, corresponding to the one-digit occupation groups in the census in Section 2 and Appendix A Table F.1. (There is still only one management task.) The 10 worker occupation groups are further categorized into low/medium/high skill tasks, or manual/routine/abstract

jobs, according to the mean wages of each occupation group. These broad skill groups summarize the underlying occupations. Employment targets use the detailed model groups, while wage targets combine individual routine occupations with broad-group means, as described below.

At the 1980 baseline, we index worker occupations by $j = 0, \ldots, 9$ and write the task technology as:

$$b(h, j) = \begin{cases} b_0 = 1 & \text{for } j = 0, \\ h - \chi_j & \text{for } j \in \{1, \ldots, 9\}, \quad 0 = \chi_1 < \chi_2 < \ldots < \chi_9. \end{cases}$$

The discrete specification replaces the continuous assignment function with task thresholds. This technology implies that, for the lowest-skill task 0, the worker's skill does not matter and everyone performs the task with equal efficiency. At the normalized baseline, all skills (h) are used in task 1; higher-order tasks have higher requirements. Away from the calibrated endpoint normalizations, including intermediate dates and projections, the lowest nonmanual requirement may differ from zero. All nonmanual tasks then use $h - \chi_j$, with requirements ordered at each date and effective skill positive on the assigned range.[21] The manual restriction has an economic role. It represents work that does not use additional worker skill, so the task has no direct upskilling channel even as its employment expands. Changes in requirements and skill supply can raise relative earnings in skill-sensitive tasks without giving manual workers the same gain. Setting the level $b_0 = 1$ only chooses units; assuming that $b(h, 0) = b_0$ for every h is the quantitative restriction. It is motivated by the low-end wage–employment pattern, not implied by that pattern. Neither general equilibrium nor the general analytical assignment model requires it. The analytical requirement comparison instead uses the explicit additive benchmark $b(h, j) = h - \chi(j)$, which remains skill-dependent at the bottom; other special benchmarks are confined to their respective propositions.

For any two nonmanual worker tasks with $\chi_{j'} > \chi_j$ and positive effective skills,

$$\frac{\partial}{\partial h} \log \frac{h - \chi_{j'}}{h - \chi_j} = \frac{\chi_{j'} - \chi_j}{(h - \chi_{j'})(h - \chi_j)} > 0.$$

Thus a simple additive requirement implements strict log-supermodularity: more-skilled workers have a comparative advantage in tasks with higher requirements. The quantitative exercise allows the requirement profile and skill distribution to change jointly. It therefore evaluates their combined effects on assignment and wages, without imposing the sufficient conditions for the isolated requirement change in Proposition 4.

The requirement ranking can change between dates. We reorder tasks when solving each equilibrium, while retaining the occupation labels and their sectoral weights. The manual/routine/abstract classification remains fixed at the 1980 grouping.

With 10 worker tasks, we solve for nine worker-task thresholds $\{\hat{h}_j\}_{j=1}^{9}$ and the managerial cutoff parameter $\hat{z}$. The discrete equilibrium conditions are summarized in Appendix E.

Bidimensional skill distribution For the quantitative analysis, we assume a skill distribution that is type IV bivariate Pareto (Arnold, 2014), with joint survival function

$$\bar{\mu}(h, z) = \left(1 + h^{1/\gamma_h} + z^{1/\gamma_z}\right)^{-a}.$$

Here a controls tail thickness and γ_h and γ_z govern the marginal skill distributions. We fix $\gamma_z = 1$ in the quantitative specification. This distribution allows the two skill densities to have different shapes. With

[21] This can be interpreted as low-order skills not being used in high-order tasks, or high-order tasks requiring a fixed cost of preparation to perform the task, resulting in less skill used in production. Both the requirement parameters and the skill distribution are recalibrated for 2019. Their joint effect on effective skills depends on occupation and selection.

	(1)	(2)	(3)
γ_m	0.362**	0.340**	0.250**
	(0.004)	(0.005)	(0.004)
ϵ	0.004**	0.002**	0.003*
	(0.000)	(0.000)	(0.001)
AIC	-573.334	-573.805	-573.527
RMSE_1	0.117	0.117	0.117
RMSE_2	0.041	0.041	0.041

Standard errors in parentheses

† $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table 4: Aggregate Production Function

The manufacturing share parameter γ_m and the manufacturing-services elasticity parameter ϵ are estimated from the time series of output and price ratios from 1947 to 2019, from the National Industry Accounts. Standard errors in parentheses. For details of the estimation, we closely follow Herrendorf et al. (2013).

$\gamma_z = 1$, the marginal managerial-skill density is monotonically decreasing; the calibrated $0 < \gamma_h < 1$ gives worker skill an interior mode. Appendix Figure F.11 shows the marginal distributions of h and z for both 1980 and 2019.

4.2 Aggregate Production Function

The aggregate production function (4) is estimated outside of the model. For the estimation, we only look at manufacturing (inclusive of mining and construction) and services (inclusive of government). We estimate the parameters γ_m and ϵ from:

$$\log\left(\frac{p_m Y_m}{PY}\right) = \log\gamma_m + (1-\epsilon)\log p_m - \log\left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right] + u_1$$

$$\log(Y) \qquad = c + \frac{\epsilon}{\epsilon-1}\log\left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}}\right] + u_2$$

where $\gamma_s \equiv 1 - \gamma_m$, c is a constant, and u_1 and u_2 are regression residuals. We estimate these equations using nonlinear seemingly unrelated regression on the annual real and nominal sectoral output series in the BEA Industry Accounts.[22] Real production by sector is computed by a cyclical expansion procedure as in Herrendorf et al. (2013) using production value-added to merge lower-digit industries (as opposed to consumption value-added in their analysis).

Sectoral prices are implied from nominal versus real sectoral quantities, which may depend on the choice of base year. For robustness, we check results using three different base years, corresponding to columns (1)-(3) in Table 4. For each column, respectively, 1947 is the first year the required data are available, 1980 is the first year in our model, and 2005 is a pre-Great-Recession base year. The values are in a similar range as in Herrendorf et al. (2013). For the calibration, we use $(\gamma_m, \epsilon) = (0.339, 0.002)$, close to the column (2) estimates of $(0.339514, 0.001947)$; Table 5 reports the values used.

The capital income share α is computed from all-sector NIPA Table 1.10, using the March 25, 2021 vintage. It is the 1980–2020 average of one minus compensation of employees divided by adjusted gross domestic income, fixed at 0.363 and assumed equal across sectors.[23] Adjusted income subtracts proprietors'

[22] The constant c is included since it is not levels but relative changes that identify ϵ.

[23] The difference between the α's when we let them differ between the two sectors was negligible. Herrendorf et al. (2015) compare this assumption against sectoral production functions that are CES in capital and labor, and find that both specifications

	Parameter	Value	Target
Fixed from data	K_{1980}	2.900	Computed from BEA/NIPA
	K_{2019}	4.104	
	α	0.363	
	γ_m	0.339	Estimated in section 4.2
	ϵ	0.002	
Fit to 1980	ν_{ij} (18)	Table 6	Within-sector employment shares by occupation
	η_i (2)		Within-sector manager share
	A_m	1.080	Manufacturing employment share
	$M_j \equiv M_0$	0.940	Output per worker, normalization
	a	9.171	
	γ_h	0.210	Relative wages by occupation
	χ_j (8)	Table 6	
Common elasticities	σ	0.620	1980–2019 output growth and within-sector employment shares
	ω	0.265	
Fit to 2019	m_j (11)	Table 6	Output per worker growth and employment shares within sectors
	a	2.706	
	γ_h	0.258	Relative wages by occupation
	χ_j (8)	Table 6	
Fixed	b_0	1.000	Not separately identified from M_0
	γ_z	1.000	Decreasing managerial-skill density
	A_s	1.000	Normalization

Table 5: Parameters

The population size is normalized, so K_t is capital per capita and is supplied from the data at each date. The elasticities σ and ω are common to 1980 and 2019. Only task productivities, (a, γ_h), and the χ_j's change between the two calibrated equilibria. Employment-share and relative-wage targets are computed from the Census and ACS using the definitions in Appendix A. Table F.1 reports the relative-wage targets at both endpoints.

income and net production taxes from gross domestic income; net production taxes are taxes on production and imports less subsidies. This all-sector model share is distinct from the sector-specific capital shares used to construct the sectoral Solow residuals. For the calibration we also need total capital stock (for manufacturing and services) for each decade, which we take from the Fixed Assets Account Table 3 and directly plug into the model.[24] Since we do not model population growth, in practice we normalize output per worker in 1980, y_{1980}, to one, and plug in $K_t = k_t/y_{1980}$ for $t \in \{1980, 1990, 2000, 2010, 2019\}$, where k_t is capital per worker in year t.

4.3 Setting Parameters

All parameters are in Table 5, except for the skill-requirement parameters χ_j, task intensity parameters (η_i, ν_{ij}), and task-TFP growth rates m_j, which are in Table 6. Below, we explain how these parameters are recovered. Appendix E has more detail.

Calibrating the distribution For given γ_h, a, and $\{\chi_j\}_{j=2}^9$, we numerically compute the thresholds $\{\hat{h}_j\}_{j=1}^9$ and $\hat{z}$ to match observed employment shares by occupation in 1980, by integrating over the skill distribution. Let w_j denote the price per unit of effective worker skill in task j, with w_0 also the manual

capture the effect of differential productivity growth across sectors equally well.

[24] Real capital stock is aggregated using the same cyclical expansion procedure used for value-added.

Ranked by mean wage (except management)	Task Wgts (ν_{ij}, η_i) Manu.	Serv.	m_j(%)	χ_j 1980	2019
Low Skill Services	**0.017**	**0.142**	**0.567**	**-**	**-**
Middle Skill	**0.816**	**0.521**			
Administrative Support	0.089	0.174	3.656	0.000	0.068
Machine Operators	0.258	0.016	6.126	0.010	0.012
Transportation	0.112	0.080	3.157	0.026	0.000
Sales	0.026	0.121	1.407	0.047	0.168
Technicians	0.034	0.040	-1.148	0.047	0.205
Mechanics & Construction	0.165	0.065	2.803	0.047	0.116
Miners & Precision Workers	0.132	0.027	6.617	0.055	0.101
High Skill	**0.167**	**0.337**			
Professionals	0.069	0.193	-1.991	0.059	0.205
Management Support	0.098	0.144	-1.815	0.091	0.267
Management	0.077	0.132	-2.676	-	-

Table 6: Calibrated Task Weights, Growth Rates and Skill Requirements

wage because $b_0 = 1$. With these thresholds, we can compute model-implied relative wages using the discrete version of (28), which is (81) in Appendix E:

$$\frac{w_1(\bar{h}_1 - \chi_1)}{w_0} = \frac{\bar{h}_1 - \chi_1}{\hat{h}_1 - \chi_1}, \quad \frac{w_z \bar{z}}{w_0} = \frac{\bar{z}}{\hat{z}}, \quad \text{and} \quad \frac{w_{j+1}(\bar{h}_{j+1} - \chi_{j+1})}{w_j(\bar{h}_j - \chi_j)} = \frac{\hat{h}_{j+1} - \chi_j}{\bar{h}_j - \chi_j} \cdot \frac{\bar{h}_{j+1} - \chi_{j+1}}{\hat{h}_{j+1} - \chi_{j+1}}.$$

Here, $\bar{h}_j$ and $\bar{z}$ denote the selected mean skills in each task. The left-hand sides are ratios of occupational mean wages, which we observe in the data. The right-hand sides depend on the thresholds and selected mean skills, which are determined by (γ_h, a, χ_j) and the employment shares. We select $(\gamma_h, a, \{\chi_j\}_{j=2}^9)$ to fit observed mean wage ratios, computing the implied thresholds $\{\hat{h}_j\}_{j=1}^9$ and $\hat{z}$ to match 1980 employment shares by occupation. At both endpoints, the wage targets are the seven routine occupations and the routine, abstract and managerial group means, each divided by the manual mean wage. Individual abstract-worker wages are not separate targets. The benchmark does not match every targeted ratio exactly.

We repeat this step using 2019 employment shares and relative wages, allowing (a, γ_h) and the requirement parameters to change. This produces separate endpoint distributions, requirements, and threshold vectors $\mathbf{x}_{1980}$ and $\mathbf{x}_{2019}$. Conditional on these endpoint values, we recover the production parameters and task-productivity growth rates so that both threshold vectors are consistent with equilibrium. Relative wages are therefore calibration targets at both dates.

Calibrated within the model We fix $\gamma_z = 1$ and normalize $b_0 = 1$, $\chi_1 = 0$ at the 1980 baseline, and $A_s = 1$. We also set $M_j = M$ for all worker and managerial tasks in 1980, since initial task-productivity levels are not separately identified from task weights in a static equilibrium. The common initial productivity M matches output per worker, while A_m matches the manufacturing employment share.

Conditional on the common elasticities and the 1980 skill distribution and requirements, we recover η_i and $\{\nu_{ij}\}_{j=0}^9$ from 1980 employment shares within each sector. There are nine independent worker-task weights per sector because $\sum_{j=0}^9 \nu_{ij} = 1$. Together with (M, A_m), these parameters fit the 1980 production and employment targets, with the employment shares shown in Figure 9.

For the 2019 equilibrium, we retain these sectoral task weights and sector-specific productivities. Task

productivities evolve according to

$$M_j(t) = M(1 + m_j)^{t-1980}, \qquad j \in \{z, 0, \ldots, 9\}.$$

Thus managerial productivity in 2019 is $M(1 + m_z)^{39}$, with the analogous expression for each worker task. The benchmark has no exogenous sector-specific productivity growth: A_m and A_s remain at their 1980 values.

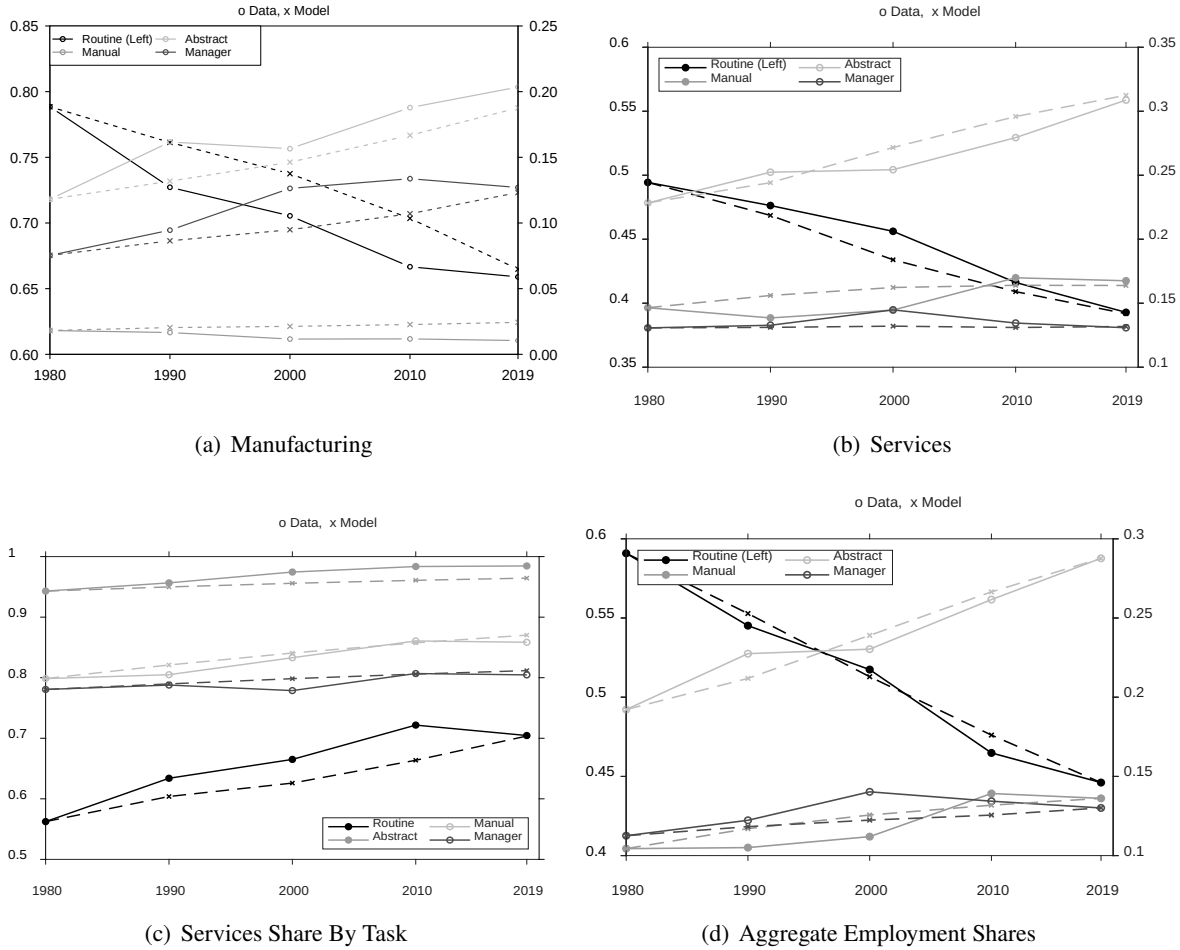
The 11 task-productivity growth rates, together with the common elasticities (σ, ω), are chosen to match aggregate output per worker growth and 2019 employment shares within sectors, conditional on the endpoint skill distributions and requirement parameters. The Pareto parameters (a, γ_h) and the χ_j's account for the changing relative-wage targets described above. Capital per worker is supplied from the data. All other parameters remain fixed across dates. The calibrated values are reported in Tables 5 and 6.

Discussion As implied by the employment data in Figure F.8(a), the manufacturing sector has higher intensity parameters for middle-skill jobs and a lower intensity for managers.[25] The productivity-growth column in Table 6 shows faster growth among the middle-skill jobs used intensively in manufacturing. With complementary sectoral outputs, this pattern provides a force toward structural change. The resulting employment response also depends on occupational composition and relative sectoral earnings, as shown in Section 3.9.

The calibrated elasticities are $\omega = 0.265 < \sigma = 0.620 < 1$, with both values held fixed across dates. Managerial and worker composites are therefore more complementary than worker tasks are to one another. A useful comparison for the elasticity across tasks is Goos et al. (2014). Their point estimate for σ is around 0.9, which is much closer to 1 than ours.[26] Their labor-demand estimates condition on occupational wages and allow for changes in industry demand. Our calibration also determines skill prices and occupational selection, and uses aggregate output growth to discipline the productivity changes. In our model, the annual direct productivity component of log task-demand ratios is $(\sigma - 1)\log[(1+m_j)/(1+m_k)]$, holding assignment thresholds fixed, with m_j the annual net productivity-growth rate. Thus faster productivity growth reduces relative task demand when $\sigma < 1$; equilibrium prices, worker reassignment, managerial selection and sectoral reallocation also affect occupational employment (equation (80)). If we were to set $\sigma = \omega = 0.9$, we would recover much higher values for m_j to explain the employment share changes in the data. The output-growth target therefore helps distinguish strong complementarity with moderate productivity growth from weaker complementarity with larger productivity changes.

5 Quantitative Analysis

The quantitative analysis asks which combinations of productivity, requirements, and skill supply account for the joint wage–employment pattern between 1980 and 2019: employment expanded at both ends of the occupational distribution, while relative wage gains were concentrated at the top. We first assess the benchmark's fit to employment and wages, then compare the mechanisms through counterfactuals, and finally connect the recovered changes to occupational task content. Endpoint wages and employment discipline the calibration; the counterfactuals and task-content associations give the selected parameter changes their economic interpretation.



(a) Manufacturing

(b) Services

(c) Services Share By Task

(d) Aggregate Employment Shares

Fig. 9: Data vs. Model, Employment Shares by Task.

The quantitative model has 11 one-digit occupation groups. For graphical representation only, we re-group the 10 worker occupations into the 3 broader categories of manual, routine, and abstract as in Table 6. Panels (a) and (b) show within-sector occupational employment shares, with routine jobs on the left axis and the other groups on the right. Panel (c) shows the fraction working in services for each occupation group; Panel (d) shows aggregate occupational employment shares.

5.1 Benchmark Fit

Figure 9 compares occupational employment shares in the model and data, grouping worker occupations into manual, routine, and abstract jobs. Within-sector employment shares discipline the calibration, conditional on the endpoint skills and requirements. Intermediate paths use the observed capital series and interpolate the changing parameters. The manufacturing employment share is matched in 1980; its subsequent change is not a separate target. The model closely matches aggregate occupational shares and the rise in the service-sector share within each occupation, consistent with the exposure mechanism in Proposition 6 and the composition effect in Lemma 3.

The paths exhibit larger routine-share declines and managerial-share increases in manufacturing than in services. They do not reproduce the post-2000 reversal in managerial employment: the model share rises from 12.2 to 13.0 percent between 2000 and 2019, whereas the observed share falls from 14.0 to 13.0 percent. The quantitative comparison concerns the long-run changes rather than the timing of this

[25] Since we normalize $M_j \equiv M$ in 1980, the parameters correspond to skill-adjusted employment weights.

[26] Since management is not a separate occupation in that paper, it has no counterpart to our ω.

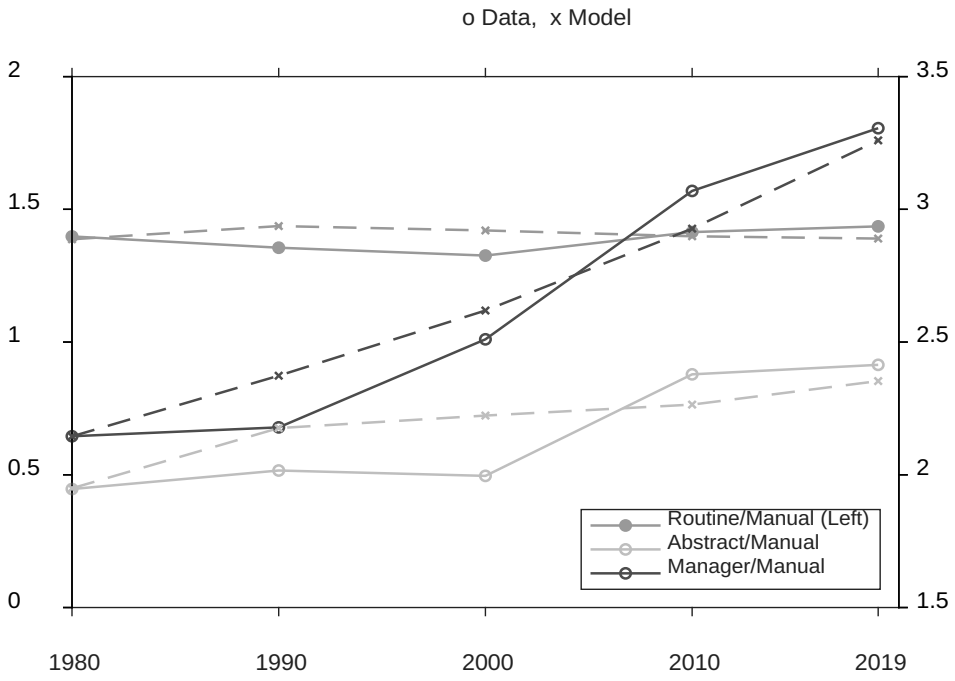


Fig. 10: Data vs. Model, Relative Occupational Wages.
Routine, abstract, and manager mean wages are each divided by the manual mean wage. Circles denote data and crosses denote the model. The routine ratio uses the left axis; the abstract and manager ratios use the right axis.

turning point.

Relative wages Relative occupational wages in 1980 and 2019 discipline the skill distribution and requirements. Figure 10 shows the broad rise in abstract and managerial wages relative to manual wages, with the largest gains for managers, though gaps remain in the abstract ratio and intermediate paths. Occupational means reflect both skill prices and selection: entry can attenuate mean wage growth if entrants have lower effective skill. Manual work is different because effective skill is constant at $b_0 = 1$, so its wage changes entirely through the equilibrium task price w_0.

Earnings at a fixed skill and occupation are common across sectors; broad-group sectoral means differ because manufacturing and services use detailed occupations in different proportions. Appendix Figure F.1 reports the within-sector wage ratios. The model abstracts from the persistent within-occupation manufacturing wage premium in the data.[27]

Appendix Figure F.2 compares within-group wage dispersion. The model reproduces its rise among routine and abstract workers and managers, but understates levels. Appendix Figure F.4 reports establishment-size patterns by sector, providing a descriptive check on the managerial margin conditional on a stable relationship between managers and establishments.

Section 5.2 next separates the contributions of task productivity, requirements, and skill supply to the joint wage–employment pattern.

[27]The sectoral wage difference is relatively stable over the observation period; see Appendix Figure F.10(b). In the discrete model, mean skills within each occupation are equal across sectors.

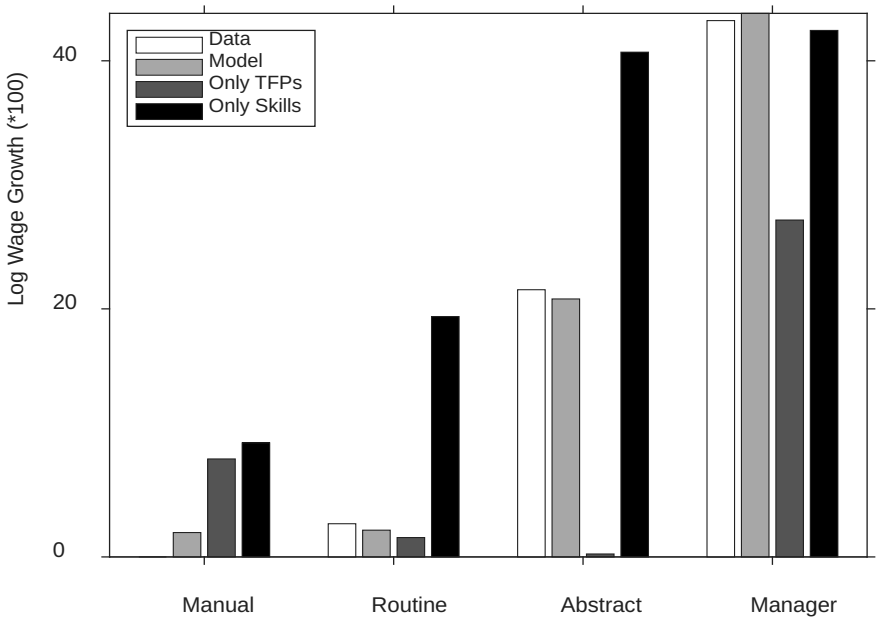


Fig. 11: Benchmark vs. Counterfactuals, Occupational Wages.
Changes in log mean occupational wages, less the observed manual-wage change, multiplied by 100; the same observed manual-wage change is subtracted from every series. The bars compare the data, full model, and the indicated combinations of productivity and skill changes.

5.2 Employment and Wage Dynamics

We use counterfactuals to separate the roles of productivity, requirements, and skill supply. The productivity-only exercise retains the calibrated task-productivity changes and holds requirements and the joint skill distribution at their 1980 values. The skill-only exercise retains the calibrated requirement and distribution changes and holds task productivities at their 1980 levels. Both use the observed capital path and the benchmark's remaining parameters. These are decompositions at the selected calibration, with no refitting of the restricted models. We first compare occupational employment and wages, then examine how task-productivity changes carry through to sectoral output and employment.

Wage growth, skill requirements, and skill supply Figures 11 and 12 should be read together. Task-productivity changes alone account for much of employment reallocation, but produce too much manual wage growth and too little abstract and managerial wage growth. Adding changes in requirements and the skill distribution brings the wage pattern closer to the data. Conversely, skill changes alone produce substantial abstract and managerial wage gains, but fail to reproduce the contraction of routine employment. At the calibrated parameters, neither force alone reproduces the joint pattern.

The managerial comparison is especially informative. Managers organize complementary worker tasks, so improvements in those tasks can raise demand for managerial services as well as managerial earnings. In the productivity-only counterfactual, this benefit brings a much larger managerial employment increase than observed over 1980–2019, even while producing too little managerial wage growth. The full model combines a smaller employment increase with larger earnings gains. Its separate managerial skill and occupational selection therefore matter for the long-run comparison, alongside the requirement channel governing wage differences among workers.

In the discrete model, mean managerial earnings relative to manual earnings satisfy $\bar{w}_z/w_0 = \bar{z}/\hat{z}$ under $b_0 = 1$ (Appendix E). The selected mean $\bar{z}$ and the relative skill price $1/\hat{z}$ can change even when

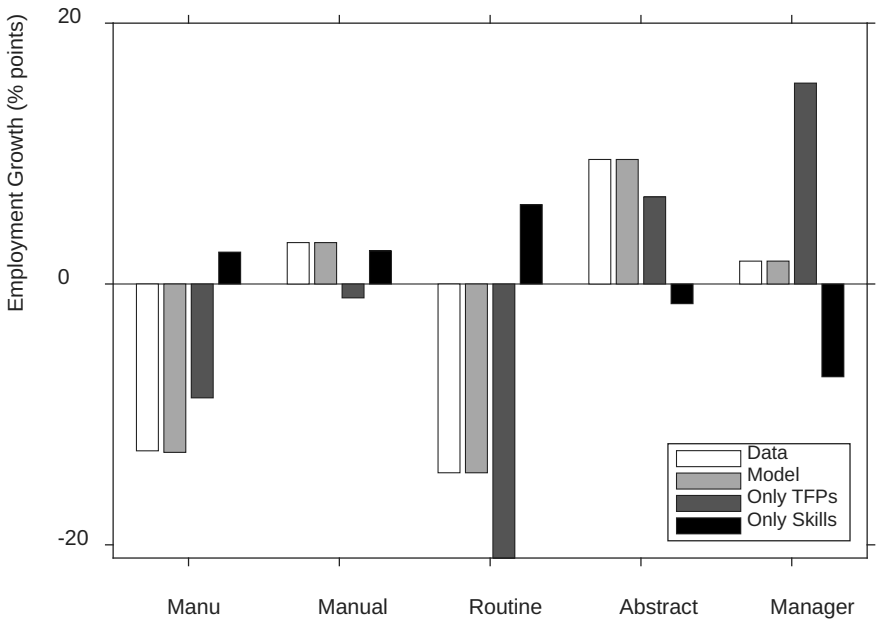


Fig. 12: Benchmark vs. Counterfactuals, Occupational Employment Shares.
Percentage-point changes in employment shares; the bars compare the data, full model, and the indicated combinations of productivity and skill changes.

managerial employment does not expand. Selection is based on managerial comparative advantage over worker earnings, rather than on h alone. This distinguishes management from both manual work, where effective skill per worker is fixed, and high-skill worker tasks, where requirements govern the return to h.

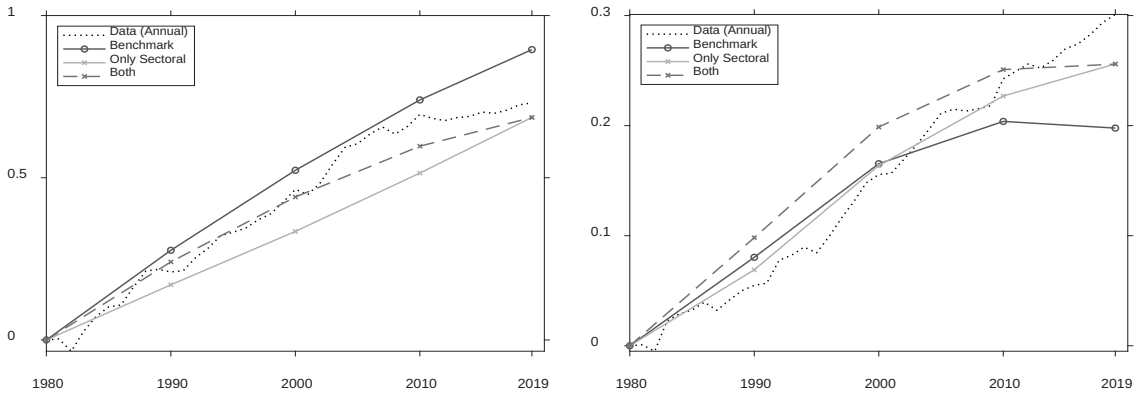
The analytical comparisons explain why the skill channel can affect managers and other high-skill workers differently. Proposition 4 gives requirement changes that favor high-h workers relative to managers of given skill, while Proposition 5 isolates the scarcity of managerial services.

The contrast between manual and abstract work explains why employment polarization need not imply comparable wage gains at both ends. Manual employment can expand as demand shifts toward complementary tasks, while its constant effective skill leaves its wage determined by the task price. Abstract wages also respond to requirements and the skills of the workers selected into those occupations. The counterfactual effects are not additive: requirements and skill supply change both selected skills and skill prices, and their effects depend on the accompanying productivity changes. Together, the experiments show why employment changes alone leave essential wage information unused.

From occupational change to sectoral change. We next analyze the role of task-specific technological change in structural change using two counterfactuals.

(1) We restrict all task-specific TFP growth to be equal, $m_j = m_z = m$ for all worker tasks j, and instead let the exogenous sector-specific TFPs (A_m, A_s) grow at rates a_m and a_s, respectively. We recalibrate common task-productivity growth and relative sector productivity to match GDP growth and the change in manufacturing TFP relative to services from 1980 to 2019. This version has common task-productivity growth and exogenous sector-specific TFP growth, but no differences in productivity growth across tasks.[28]

[28] Sectoral TFP is constructed from the NIPA accounts. Real value-added and capital are computed via cyclical expansion from the industry accounts, labor is computed from full-time equivalent persons in production in NIPA Table 6, and TFP is the Solow residual by sector.



(a) Log TFP, Manufacturing

(b) Log TFP, Services

Fig. 13: Benchmark vs. Counterfactuals, Sectoral TFP.
Data: NIPA. Log 1980 levels are normalized to 0, so the slopes of the lines are the growth rates.

(2) We allow both exogenous task- and sector-specific TFP growth, and recalibrate $\{m_z, m_0, \ldots, m_9\}$ and relative sector productivity to match the change in occupational employment shares, GDP growth, and relative manufacturing-to-services TFP growth from 1980 to 2019. Recall that our benchmark calibration of Section 4 restricted a_m and a_s to be 0 but did not target sectoral TFPs.

In both cases, we keep all other parameters at their benchmark values of Tables 5–6, and only recalibrate productivity growth. We normalize $A_s = 1$: task productivity carries the common scale and A_m the relative sectoral shifter. The absolute sectoral TFP paths are model outcomes rather than separate targets.

We focus on relative sectoral TFP as one channel of structural change, whether its growth is exogenous (caused by a_m and a_s) or endogenous through task composition (as in Section 3.9). Occupational assignment and selection also change relative sectoral average earnings, which enter the headcount relationship in (23). The counterfactual paths are compared over the same 1980–2019 interval as the benchmark.

TFP and output growth Figure 13 shows the paths of sectoral log TFPs in the data, in our benchmark calibration, and in the two counterfactuals. The benchmark and counterfactuals match observed GDP growth between the two calibrated endpoints. Their common capital path also implies the same model aggregate TFP growth, which need not equal the separately measured aggregate TFP series (Appendix Figure F.12).[29]

The benchmark generates faster TFP growth in manufacturing than in services, although it overstates manufacturing growth and understates service growth relative to the data. The calibrated task-productivity growth rates are higher among routine jobs, on which manufacturing is more reliant. Proposition 6 gives sufficient conditions under which greater routine exposure produces both higher relative manufacturing TFP and a declining employment share. Since the benchmark does not target relative sectoral TFP growth, these paths provide a further comparison with the data.

Allowing exogenous sectoral productivity growth brings the relative TFP change into line with the data by construction. Appendix Figure F.13 reports the corresponding sectoral output paths. The question is whether the improved relative TFP fit also improves the employment outcomes. Capital allocation,

[29]Denoting aggregate TFP as Z_t, since $Y_t = Z_t K_t^{\alpha}$ (labor is normalized to one) and we plug in the empirical values of K_t for all calibrations, matching GDP fixes model aggregate TFP given this capital path. Equality with an independently constructed empirical TFP series additionally requires consistent measurement and capital shares.

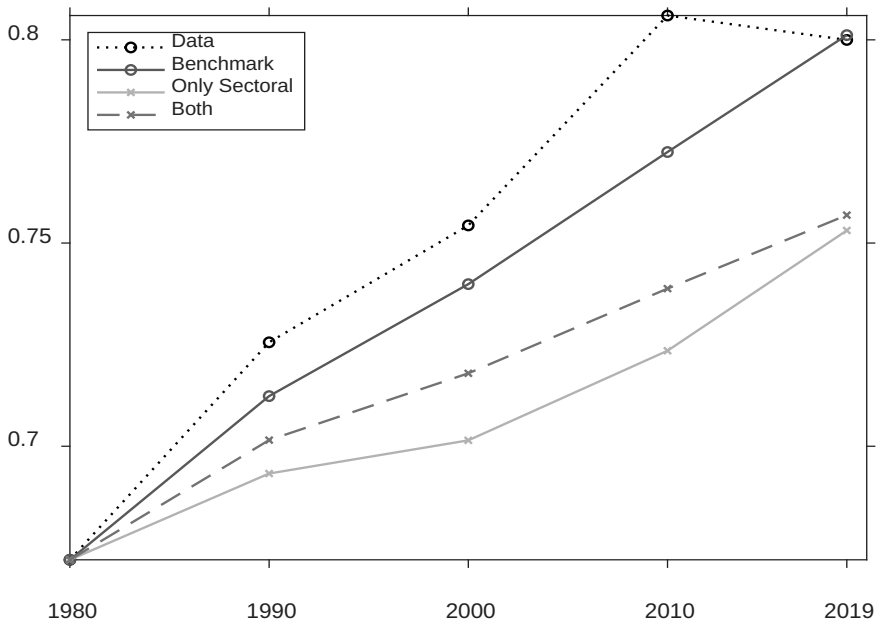


Fig. 14: Benchmark vs. Counterfactuals, Service Employment Share.
Vertical axis is the services share of overall employment.

occupational composition, and average earnings all enter the relationship between output and headcounts in (23).

Structural change and polarization Figure 14 shows that both counterfactuals generate less reallocation toward services than the benchmark. Their occupational implications differ sharply (Appendix Figures F.14 and F.18). With common task-productivity growth, the routine employment share rises and managerial employment falls, reversing the observed changes. Allowing task-specific growth restores the occupational changes, which this second counterfactual targets, but still produces too little sectoral reallocation. Matching relative sectoral TFP therefore does not suffice to match employment across both occupations and sectors. Relative average earnings also matter for the headcount relationship in (23).

The comparison separates occupation-specific from sector-specific productivity changes. Lemma 3 further isolates the pure between-sector composition contribution; it does not restrict the endogenous within-sector responses to a sectoral shock. Section F.3 illustrates this accounting distinction using the 1980 and 2019 employment shares. The main quantitative question remains how productivity, upskilling, and skill supply jointly account for wages and employment.

5.3 Task Content, Productivity, and Wage Growth

The analytical and counterfactual results distinguish task productivity from changes in the use and supply of skills. We now ask whether the recovered changes correspond to different observable characteristics of work. Figure 16 relates them to employment and wages; Figure 17 relates them to task content and educational attainment. The task-content and education comparisons provide external evidence for interpreting the calibrated channels; employment and wages are already used in the calibration.

Figure 16(a) shows a correlation of -0.913 between task-productivity growth and employment-share changes. The relevant question is what kind of technological change those fitted productivity gains represent. We compare them with routine-task measures and then contrast the result with the effective-skill

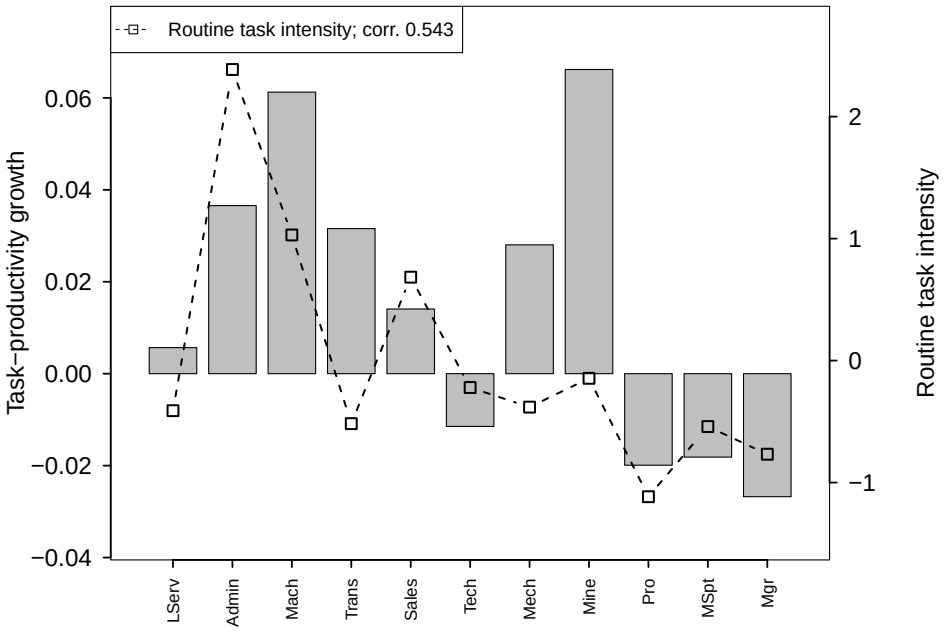


Fig. 15: Task-Specific Productivity Growth and Routine Task Intensity.
Calibrated task-specific productivity growth and the DOT-based routine task intensity (RTI) index across the 11 occupation groups. The two series use separate vertical axes. Here RTI is computed from broad-group mean task scores; Appendix Table F.3 instead uses the group average of detailed-occupation RTI.

channel.

The aggregate routine task intensity (RTI) measures provide an initial comparison. Figure 15 uses the DOT-based measure in Autor and Dorn (2013), constructed from the task indices of Autor et al. (2003). We also consider the O*NET-based measure in Acemoglu and Autor (2011). Productivity growth is positively correlated with both, but their explanatory power is limited (Appendix Table F.3). The disaggregated task measures below are more informative.[30]

Routine-manual content is positively associated with task-productivity growth, while nonroutine manual-interpersonal content is negatively associated. The respective univariate regressions have R^2 values of 0.660 and 0.526 (Appendix Table F.3); Figure 17(a) shows the routine-manual comparison. The recovered productivity changes thus line up more closely with particular activities than with an aggregate routine index. They support a routine-automation interpretation of the employment channel.[31]

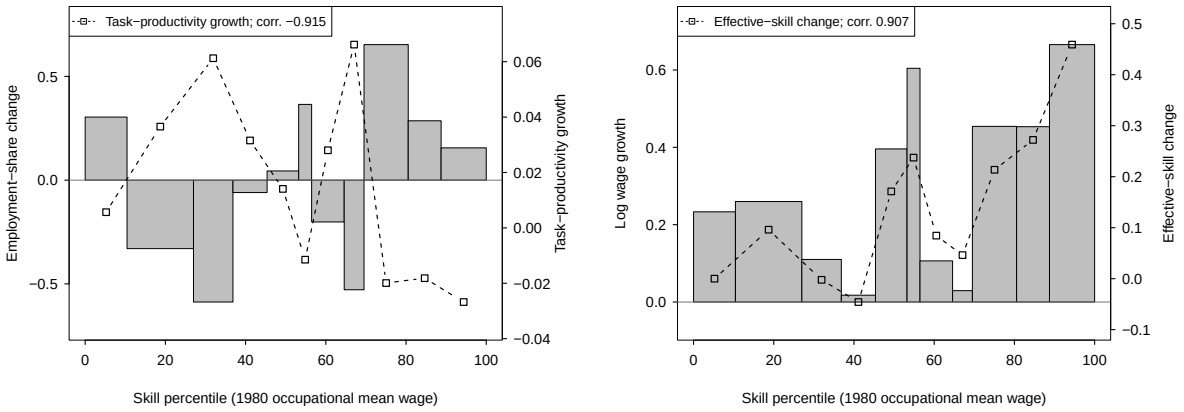
Task productivity can also reflect forces omitted from the model, including offshoring and differences in capital–skill substitutability.[32] The task-content comparison gives the fitted employment channel an economic interpretation without attributing every productivity gain to automation.

Task-productivity growth chiefly captures the reallocation associated with firms' demand for task output. To summarize the skill channel, let $\bar{h}_t = \mathbb{E}h_t$ denote unconditional mean worker skill at date t,

[30]Appendix Figure F.8(b) reports the concentration of high-RTI occupations in the middle of the wage ranking, following Autor and Dorn (2013).

[31]Task productivity here is factor-neutral. A capital-augmenting improvement, such as a fall in task-specific capital costs, can generate a similar reallocation channel; see Section 3.6 and Goos et al. (2014).

[32]Task-specific capital–labor substitution differs from an elasticity that varies directly with worker skill, as in Krusell et al. (2000). Cheaper foreign labor can have a task-productivity-like effect; the calibrated rates have only a weak association with the offshorability measure of Firpo et al. (2011).



(a) Task TFP Growth and Employment Shares

(b) Effective Skills and Wage Growth

Fig. 16: Calibrated Parameters vs Data

Panel (a) compares calibrated task-productivity growth with employment-share changes. Panel (b) compares occupational mean wage growth with Δ_χ, the signed log effective-skill index defined in the text. The series use separate axes.

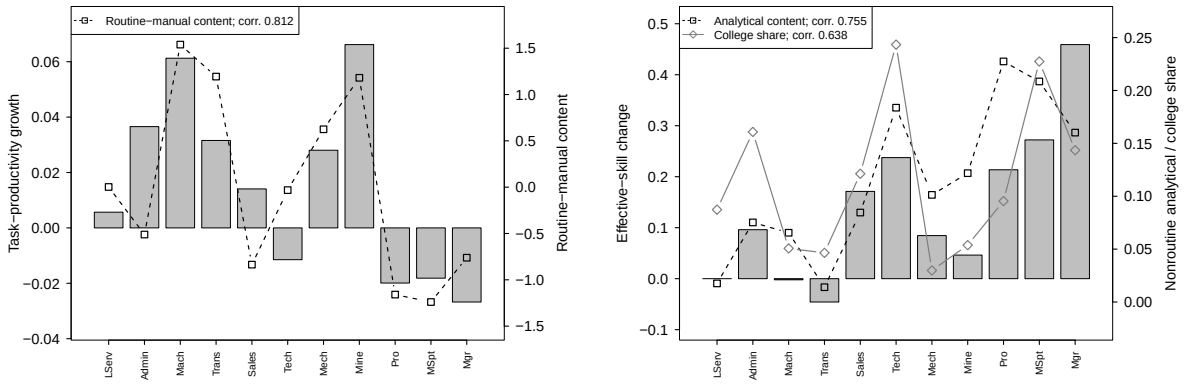
and define, for each nonmanual worker task j,

$$\Delta_{\chi,j} \equiv \log\left[1 - \frac{\chi_{j,1980}}{\bar{h}_{1980}}\right] - \log\left[1 - \frac{\chi_{j,2019}}{\bar{h}_{2019}}\right].$$

The index is positive when effective skill evaluated at the population mean, relative to that mean, falls. It combines changes in requirements and the skill distribution. We suppress the occupation subscript when referring to the index across groups. The manual index is fixed at zero, consistent with its constant effective input. For managers, we replace χ_j by the normalized joint-skill statistic $C_z = \mathbb{E}[z \mid h \geq \bar{h}]/a$ in the same transformation. This managerial entry is a distribution statistic, not a task requirement or the mean skill of selected managers. Appendix F.2 gives its construction and role in the AI scenarios.

The college-share comparisons help distinguish the two channels. In the 1980–2019 regressions in Appendix Table F.3, task-productivity growth is negatively associated with the change in the college share in a univariate specification; that coefficient is not significant after routine-manual content is included. By contrast, Appendix Table F.2 shows a positive association between the effective-skill change and the college-share change, alongside its association with nonroutine analytical content. These patterns motivate separate productivity and skill channels without making educational attainment a direct measure of either primitive. A common technological advance could change both task productivity and occupational skill requirements, with the two sets of task measures capturing different dimensions of that change.

The joint index has a correlation of 0.907 with occupational wage growth in Figure 16(b), a relationship partly disciplined by the wage targets. Its correlations with nonroutine analytical content and the college-share change are 0.755 and 0.638 in Figure 17(b). These external associations become stronger when managers are excluded and remain positive when any one occupation group is omitted (Appendix Table F.4). The contrast with routine-manual productivity growth is clear: the task characteristics associated with the movement of jobs differ from those associated with the changing rewards to skills. This descriptive evidence complements the counterfactuals' separation of the two channels.



(a) Task-specific TFP Growth and O*NET

(b) Δ_χ, O*NET and College

Fig. 17: Task Productivity, Effective Skills, and Task Content

Panel (a) compares task-productivity growth with routine-manual task content. Panel (b) compares Δ_χ with nonroutine-cognitive analytical content and the change in the college-educated share. Legend entries report correlations; the series use separate axes.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Δ_{2010} Wage	-0.201*	-0.201*	-0.233*	-0.208*				
	(0.086)	(0.087)	(0.090)	(0.082)				
NR-Cog Anal	0.015	0.015	0.008		0.002	0.003	-0.003	
	(0.008)	(0.008)	(0.008)		(0.007)	(0.007)	(0.008)	
AI (Webb)		0.000				-0.000		
		(0.006)				(0.007)		
AIOI (Felten et al.)			0.015**	0.018**			0.008	0.006
			(0.006)	(0.006)			(0.006)	(0.005)
Constant	0.048**	0.049**	0.051**	0.049**	0.034**	0.034**	0.034**	0.034**
	(0.008)	(0.008)	(0.008)	(0.008)	(0.006)	(0.006)	(0.006)	(0.006)
R^2	0.030	0.030	0.042	0.039	0.001	0.001	0.004	0.004

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$

Table 7: AI Exposure and Occupational Wage Growth, 2010–2019.
The dependent variable is the change in log occupational mean wages over 2010–2019. Columns (1)–(4) control for the corresponding 2000–2010 log change, denoted Δ_{2010}; columns (5)–(8) omit this control. Exposure and task-content regressors are standardized. Regressions are unweighted across occupations and use heteroskedasticity-robust standard errors. Exposure measures are from Webb (2020) and Felten et al. (2019). Coefficients are descriptive associations.

6 AI and Other Applications

6.1 Artificial Intelligence, Wages, and Employment

AI raises the same joint question as the historical analysis: which activities become more productive, and how does this change the return to the skills used in them? Its exposure pattern need not resemble routine-manual automation. We first examine occupational evidence for 2010–2019, then compare two exposure-based scenarios to distinguish wage changes from employment-share changes.

Recent occupational evidence Table 7 reports the wage-growth regressions. Conditional on the preceding wage change, the artificial intelligence occupational impact (AIOI) measure of Felten et al. (2019) is positively associated with wage growth: its coefficients are 0.015 and 0.018 in columns (3) and (4). The association is not statistically significant in the specifications without that control, and the Webb AI measure is not significantly associated with wage growth in either specification. The earlier nonroutine-analytical association is also weaker over this shorter period than in Table 3.

These results do not establish that AI caused the observed wage changes. They show that the relationship depends on both the exposure measure and the specification. In particular, an index measuring exposure need not measure displacement, and a positive wage association cannot by itself distinguish productivity gains, changing skill requirements, and selection.

Table 8 reports employment-share changes. AIOI has no statistically significant association in the reported specifications, whereas Webb's AI measure has a positive association. Thus the wage association of AIOI is not accompanied by a corresponding employment-share result, and the employment association differs across exposure measures.

An illustrative AI scenario The historical decomposition and task-content comparisons suggest examining AI through both productivity and skills. To isolate the latter channel, the scenarios change

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Δ_{2010} Share	0.204*	0.226**	0.198*	0.198*				
	(0.079)	(0.084)	(0.084)	(0.084)				
R-Manual	-0.019	-0.023	-0.028	-0.028	-0.042	-0.048*	-0.060*	-0.060*
	(0.023)	(0.023)	(0.030)	(0.030)	(0.024)	(0.024)	(0.029)	(0.029)
AI (Webb)		0.083**				0.077**		
		(0.027)				(0.027)		
AIOI (Felten et al.)			-0.013	-0.013			-0.027	-0.027
			(0.045)	(0.045)			(0.044)	(0.044)
Constant	0.030	0.030	0.030	0.030	0.027	0.027	0.027	0.027
	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)
R^2	0.035	0.077	0.035	0.035	0.011	0.048	0.014	0.014

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$

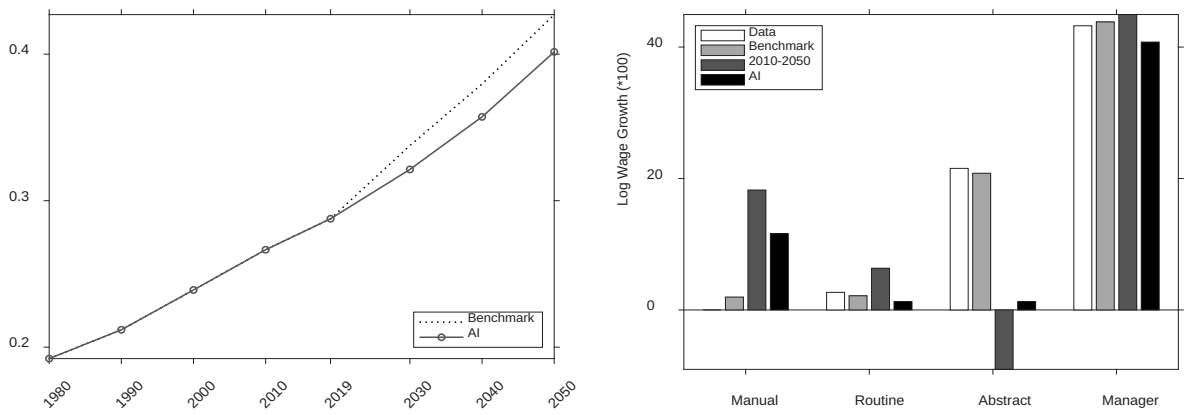
Table 8: AI Exposure and Occupational Employment-Share Changes, 2010–2019.
The dependent variable is the proportional change in the occupational employment share, $s_{2019}/s_{2010} - 1$, rather than a percentage-point change. Columns (1)–(4) control for $s_{2010}/s_{2000} - 1$, denoted Δ_{2010}; columns (5)–(8) omit this control. Exposure and task-content regressors are standardized. Regressions are unweighted across occupations and use heteroskedasticity-robust standard errors. Exposure measures are from Webb (2020) and Felten et al. (2019). Coefficients are descriptive associations.

the joint skill index while relative task productivities follow their historical continuation paths. The exposure-to-effective-skill mapping is imposed rather than estimated from the wage regressions.

Recent evidence makes this distinction consequential. Noy and Zhang (2023) find productivity gains from generative AI in professional writing tasks, and Brynjolfsson et al. (2025) document gains in customer support that are particularly large for less-experienced and lower-skilled workers. These findings show that AI assistance can narrow performance differences, so they do not identify a uniform requirement shock. In Althoff and Reichardt (2026), task simplification is the key channel through which AI reduces wage inequality by allowing lower-skill workers to compete for previously inaccessible jobs. This mechanism motivates attention to requirements, but does not identify the direction or size of our exposure-based scenario changes.

Starting from 2019, the scenario changes $q_j = \log(1-\chi_j/\bar{h})$ in proportion to standardized occupational exposure; its historical change is $-\Delta_{\chi,j}$. The manual index stays at zero. For workers, the change operates through occupational requirements. Management has no analogous additive requirement: its index $q_z = \log(1 - C_z/\bar{h})$ instead changes the joint skill distribution and hence the supply and selection of managerial ability. Mean worker skill follows the same historical extrapolation in the scenario and continuation benchmark. A scalar intercept specifies the common component of the nonmanual and managerial index changes, which relative exposure alone does not determine. A separate common task-productivity level matches continuation GDP in 2030. Appendix F.2 gives the mapping and continuation rules.

Figure 18 illustrates the distinction using the broad-AI exposure measure of Felten et al. (2019). Abstract employment shares rise in both paths, but rise less under AI than under the continuation benchmark. The AI path also gives higher abstract relative-wage growth than the projected 2010–2050 benchmark, while managerial relative-wage growth is lower than in that benchmark. These comparisons are within-occupation differences between the two projected paths. The plotted wage bars use a common historical normalization, so their position relative to zero should not be interpreted as an absolute wage gain or loss.



(a) Abstract Employment Share

(b) Occupational Wage Growth

Fig. 18: Wages and Employment in an Illustrative AI Scenario.

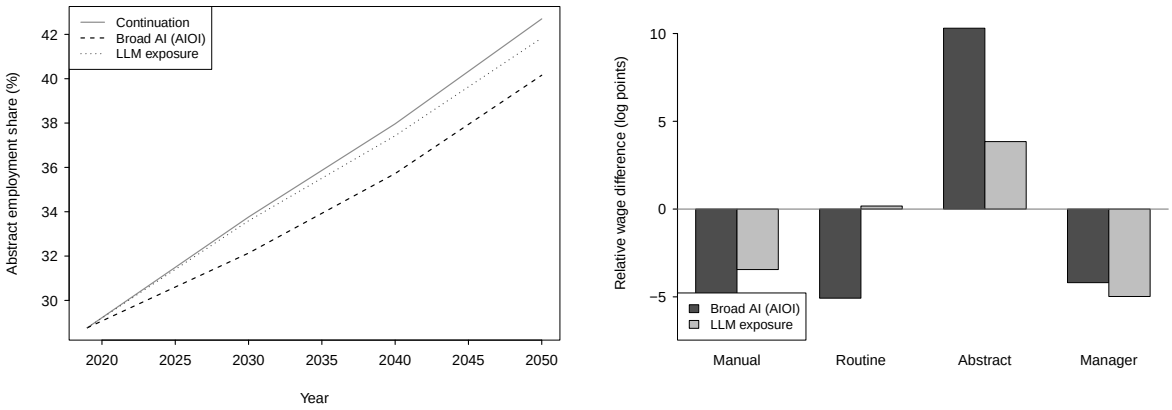
Panel (a) compares abstract employment shares under the continuation benchmark and AI scenario. Panel (b) compares log growth in occupational mean wages relative to the economy-wide mean wage, multiplied by 100, after subtracting historical manual relative-wage growth from every bar. The Data and Benchmark bars describe the historical comparison; the 2010–2050 and AI bars compare the projected continuation and AI scenarios. Interpret AI effects as differences between the last two bars within each occupation, rather than relative to zero.

Exposure measures and distributional incidence We compare the broad-AI measure with the human-rated β measure of Eloundou et al. (2024). The AIOI measure links progress in AI applications, including image recognition, translation and strategic games, to workplace abilities and then occupations. The LLM measure instead assesses whether assistance can halve the time needed for a task at comparable quality. Its $\beta = E1 + 0.5E2$ combines the share of tasks exposed to direct LLM assistance, $E1$, with half the share exposed when additional software is available, $E2$. These measures differ in technology scope, unit of measurement and assessment method, rather than being successive vintages of the same index.

We aggregate both measures with 2010 labor-supply weights and occupational definitions, including self-employed people in the manager group, and standardize with equal weights across the eleven groups. The measures are highly correlated across these groups (Pearson 0.938), but differ in economically relevant ways. Under LLM exposure, administrative support becomes the most exposed group, ahead of management support; standardized exposures of abstract workers and managers are lower, while sales exposure is higher. These are differences on a relative index scale, not percentages of workers displaced. Appendix F.2 reports coverage, exposure vectors and detailed correlations.

Both scenarios use the same functional mapping, historical dispersion loading and continuation rules. They differ in the exposure vector and the scalar skill intercept: 0.470 for broad AI and 0.390 for LLM exposure. These intercepts are scenario assumptions; matching GDP does not identify them because a separate common task-productivity level sets output in 2030. Figure 19 shows the shared abstract-worker response: employment is lower and relative wages are higher than under continuation. In 2050, abstract employment is 0.86 percentage points below continuation under LLM exposure, compared with 2.54 points under broad AI; abstract mean wages relative to the economy-wide mean are higher by 3.84 and 10.30 log points, respectively. The effective-skill changes affect both the workers selected into abstract occupations and their relative earnings, allowing slower employment growth to coexist with faster wage growth.

The detailed incidence differs across the two scenarios. Managerial relative mean wages are lower in both, while managerial employment is 0.43 percentage points lower under broad AI and 0.29 points higher under LLM exposure. Routine employment rises relative to continuation in both scenarios, but



(a) Abstract Employment Share

(b) Relative Wages in 2050

Fig. 19: AI Exposure Measures, Employment and Relative Wages.

Panel (a) compares abstract employment shares under continuation, the broad-AI scenario (AIOI) and the human-rated LLM-exposure scenario (β). Panel (b) reports 2050 differences from continuation in log occupational mean wages relative to the economy-wide mean, multiplied by 100. Within-occupation subtraction removes the common historical normalization used in Figure 18. Both exposure scenarios start from the calibrated 2019 baseline and match continuation GDP in 2030.

routine relative wages rise under LLM exposure and fall under broad AI. Table F.6 reports the 2030 and 2050 levels. The exposure vectors and skill intercepts vary together, so differences between the scenarios cannot be attributed to the exposure measure alone. Matching GDP in 2030 makes their distributional incidence comparable at that output level; GDP need not agree thereafter.

Both scenarios are conditional projections from the 2019 economy. The LLM ratings reflect capabilities assessed in 2023–2024; technical time savings need not translate into adoption or displacement. Neither measure identifies a requirement shock or the balance between productivity and requirements, and aggregation loses within-group detail. The shared abstract-worker response illustrates the paper's central distinction: employment exposure alone does not determine wage incidence when technology also changes the returns to skills and occupational selection.

τ_K (%)	Y Loss (%)	T/Y_0 (%)	K_R/K (%)	L_R (%)
		1980		
0	-	-	53.22	59.08
10	2.86	1.72	51.31	59.49
20	5.41	3.09	49.56	59.86
30	7.73	4.21	47.95	60.20
40	9.83	5.12	46.46	60.52
50	**11.75**	**5.89**	**45.08**	**60.81**
		2019		
0	-	-	33.39	44.61
10	1.80	1.09	31.67	44.98
20	3.44	1.99	30.14	45.32
30	4.94	2.74	28.77	45.64
40	6.31	3.37	27.54	45.93
50	**7.59**	**3.91**	**26.42**	**46.21**

Table 9: Taxing Capital Used in Routine Tasks

τ_K is the tax rate on routine-task capital, T/Y_0 is tax revenue as a share of baseline output, and K_R/K and L_R are routine-task capital and employment shares. Output losses are relative to the no-tax equilibrium in the same year. All entries are percentages.

6.2 Robot Taxes

The AI comparison shows that employment and relative wages can respond differently to the same technological change. We now turn from the direction of technology to policies intended to preserve particular jobs. The first asks whether taxing capital used in routine tasks can preserve routine employment. Table 9 reports this experiment separately at the calibrated 1980 and 2019 equilibria. Here "robots" refer to capital assigned to routine tasks, rather than a separately modeled technology or adoption decision.

The experiment is motivated by the robot-tax question studied by Guerreiro et al. (2022). Subsequent work makes clear why employment protection alone does not determine the desirable tax. Thuemmel (2023) studies robot taxation alongside nonlinear income taxation when automation changes wage gaps among manual, routine and cognitive workers. Costinot and Werning (2023) relates technology regulation to its distributional incidence and the available redistributive instruments. We report allocation and output effects of a routine-capital tax; evaluating its optimal rate would require the welfare and tax-system structure used in that literature.

The tax raises the user cost of capital in routine tasks from R_0 to $R_0(1+\tau_K)$. With Cobb–Douglas capital within each task, this is equivalent in unit costs to replacing M_j by $M_j(1+\tau_K)^{-\alpha}$, without changing physical productivity. Equilibrium task prices and occupational assignment then adjust. Aggregate capital also adjusts at the unchanged pretax rental rate R_0, so the reported output loss combines reallocation with a contraction in capital. A higher routine employment share is therefore an equilibrium allocation response in this calibration; it does not measure the welfare gain of the workers initially in routine jobs.

In the reported 2019 experiment, a 50 percent tax raises the routine employment share from 44.61 to 46.21 percent and lowers output by 7.59 percent. The corresponding output loss in the 1980 economy is 11.75 percent, with the routine employment share rising from 59.08 to 60.81 percent. Thus protecting an occupational employment share can be costly in output, and the trade-off differs between the two calibrated economies. Both productivities and skills differ across those economies; this comparison alone does not isolate the contribution of upskilling. Training and retirement responses are outside this static comparison.

6.3 Manufacturing Subsidies

The robot tax directly targets capital in routine tasks. A manufacturing subsidy instead targets the sector in which routine work is concentrated. The historical decomposition suggests a reason to distinguish the two: manufacturing decline and routine employment decline are connected through task composition, but neither is an accounting measure of the other.

Recent industrial-policy research examines the market failures that can justify sectoral support, including sectoral economies of scale (Juhász et al., 2024; Bartelme et al., 2025). Our exercise asks a narrower question: how much routine employment is recovered by increasing manufacturing's employment share? It holds technology and requirements fixed within each calibrated economy and includes no learning, innovation externalities or trade. The financing rates below describe the cost of this particular employment target.

We consider an illustrative subsidy financed by a tax on services. Let $\kappa = L_s/L_m$ and $r = \bar{w}_s/\bar{w}_m$, so $K_s/K_m = \kappa r$. With total employment normalized to one,

$$L_m = 1/(1+\kappa),\ L_s = \kappa/(1+\kappa). \tag{37}$$

From (37), if the manufacturing employment share increases by x (measured as a fraction of total employment), κ must change to

$$\kappa' = \frac{\kappa - x(1+\kappa)}{1 + x(1+\kappa)}.$$

Let τ_s be the tax rate on services and τ_m the manufacturing subsidy rate. Purchasers face prices $(1+\tau_s)p_s$ and $(1-\tau_m)p_m$, respectively. Using primes for the policy equilibrium, (23) then changes to

$$\kappa' = \frac{L_s'}{L_m'} = \frac{\gamma_s}{\gamma_m}\left(\frac{\Phi_s'}{\Phi_m'}\right)^{\epsilon-1}(r')^{-[\alpha+\epsilon(1-\alpha)]}\left(\frac{1-\tau_m}{1+\tau_s}\right)^{\epsilon}$$

while budget balance requires

$$\tau_s p_s Y_s = \tau_m p_m Y_m \quad \Rightarrow \quad \tau_s = \tau_m \cdot \frac{p_m Y_m}{p_s Y_s} = \tau_m/(\kappa' r').$$

Combining these equations gives the tax and subsidy rates for a specified employment-share increase x. Table 10 reports $x = 0.05$ and $x = 0.10$ for the 1980 and 2019 economies. In 2019, a 5-percentage-point increase in manufacturing employment raises the routine employment share by only 1.22 percentage points. The reason is occupational composition: manufacturing also employs nonroutine workers, while routine jobs are used in services as well. Targeting a sector and targeting an occupation are different policies. The subsidy shifts purchasers' demand toward manufacturing, but some of the resulting jobs are managerial, abstract or manual, while the service tax also affects routine jobs in services. Moreover, sectoral average earnings change with assignment. A given change in headcounts therefore does not imply the same change in the tax base: the budget depends on $\kappa' r'$, not κ' alone. Comparing 1980 and 2019 also changes both skills and productivity, so differences in the financing cost cannot be attributed to upskilling alone.

6.4 Work from Home

The policy exercises hold technology fixed and change relative prices. As a final illustration, we return to the direction of technological change, examining task-demand shifts associated with work from home and infectious exposure. WFH has persisted beyond the pandemic (Barrero et al., 2023); we use it here to

L_m (%)		τ_s (%)		L_R (%)	
1980	2019	1980	2019	1980	2019
32.79	19.88	-	-	59.08	44.61
+5		60.64	31.29	60.60	45.83
+10		**74.84**	**40.33**	**62.11**	**47.06**

Table 10: Manufacturing Subsidies in the 1980 and 2019 Economies

Columns compare the calibrated 1980 and 2019 economies. The first row gives no-policy employment shares. Subsequent rows target increases of 5 and 10 percentage points in manufacturing's employment share; τ_s is the required service tax and L_R is the aggregate routine employment share. Employment shares and tax rates are in percent. Sectoral average earnings and the balanced-budget tax are determined jointly with the policy equilibrium.

motivate a different direction of task-productivity growth.

Figure 20 reports pre-pandemic WFH and infectious-exposure measures.[33] We redirect task-productivity growth toward occupations with less work performed at home or greater exposure to infection. Both scenarios start from the calibrated 2019 economy, retain the same capital path, and extrapolate historical changes in skill distributions and requirements. A common intercept in annual task-productivity growth matches continuation-benchmark GDP in 2030; these scenario growth rates continue thereafter. This differs from the AI exercise, where the GDP adjustment is a common productivity level reached by 2030.

Figure 21 shows less reallocation toward services under both scenarios than under continuation, with a much larger difference under infectious exposure. Both also leave a larger routine employment share (Figure 22). The managerial responses go in opposite directions: the exposure scenario lowers managerial employment relative to continuation, while the WFH scenario raises it. Output matches continuation in 2030 but grows more slowly thereafter under both scenarios.

These differences operate through task costs and complementary task demand. Faster productivity growth in occupations with little observed WFH or greater infection exposure need not raise their employment shares: fewer workers may be required to supply their task output, while demand expands for complementary tasks. Manufacturing and services use different occupational mixes, so a sector-neutral change in task productivity also reallocates employment between sectors. The contrasting managerial responses reflect the different incidence of the two productivity changes across managerial and worker tasks. The common capital path and continuation of skills isolate the direction of productivity change; the GDP normalization removes the output difference only in 2030.

These exercises illustrate how the direction of task-level change affects employment across occupations and sectors. They are conditional comparisons rather than estimates of realized pandemic effects.

[33] Research on WFH feasibility and occupational exposure to social distancing provides related measurement context (Dingel and Neiman, 2020; Mongey et al., 2021). Our indices instead use observed time at home and O*NET infection and proximity measures. Li et al. (2026) study a different WFH margin: wage differences between remote and on-site workers within occupations, with a substantial role for selection. We do not model the choice to work from home; our scenarios concern reallocation across occupations, rather than that within-occupation wage premium.

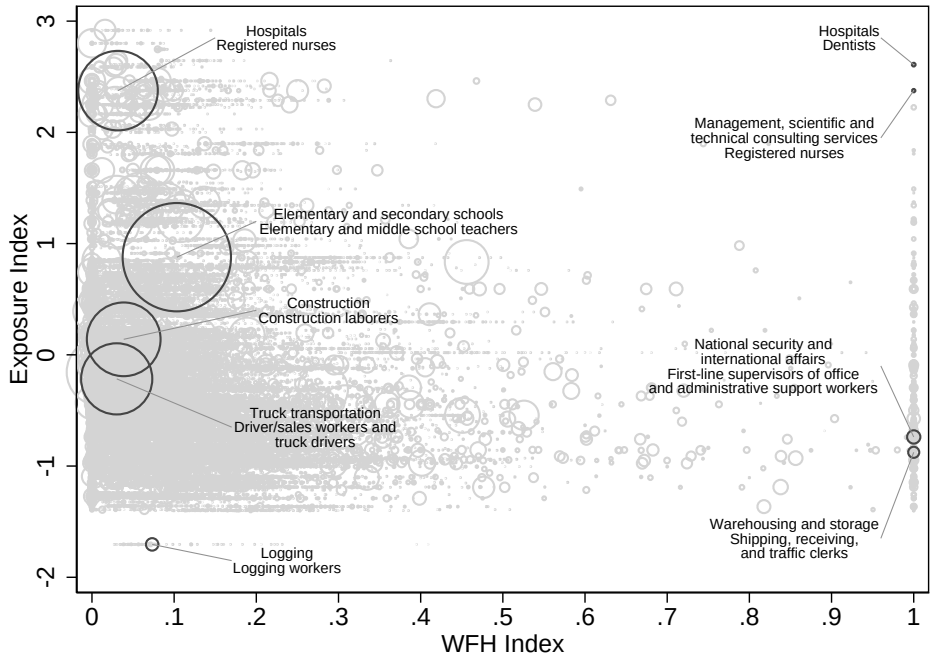
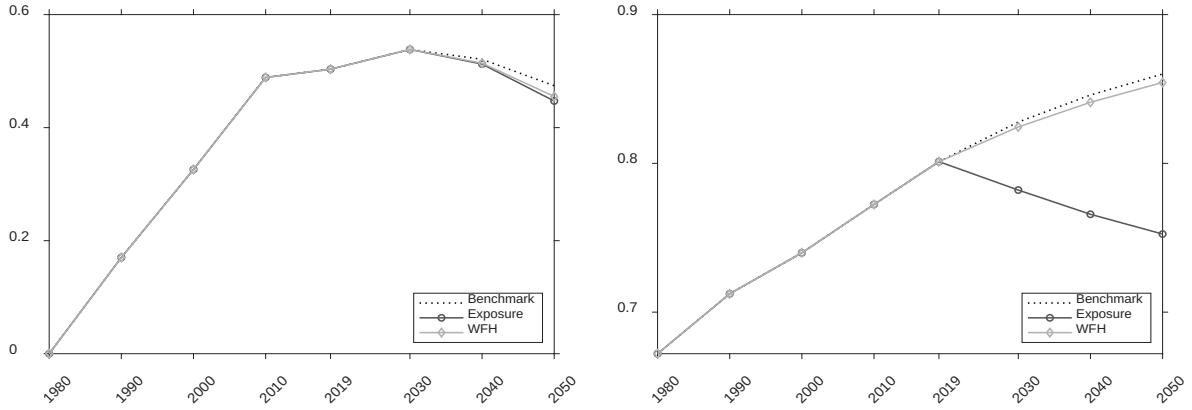


Fig. 20: Infectious Exposure and Work-From-Home Indices.

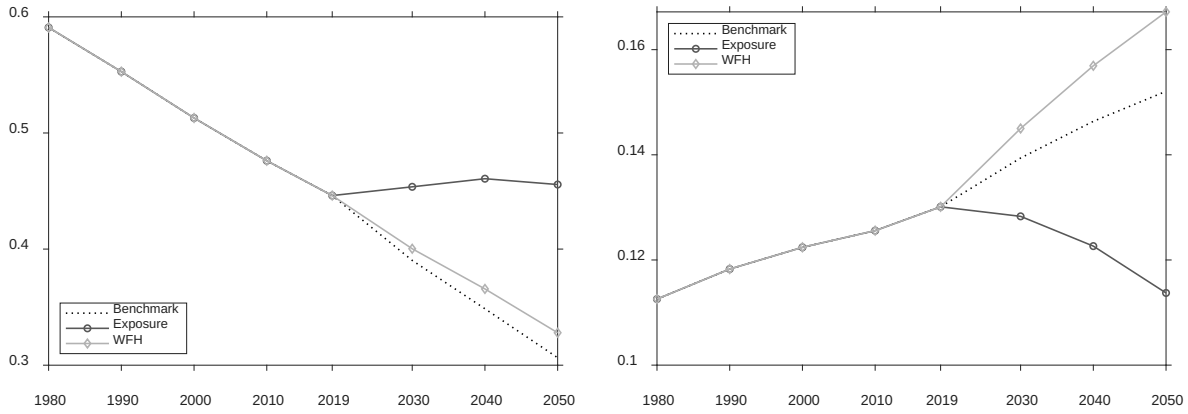
Work from home is measured using the 2014–2018 American Time Use Survey. Infectious exposure averages standardized O*NET measures of disease exposure and physical proximity. The figure aggregates occupation–industry cells using labor-supply weights; circle sizes reflect employment. These pre-pandemic indices determine the direction of the scenario changes.



(a) log GDP (b) Structural Change

Fig. 21: Work-from-Home and Exposure Scenarios: Output and Structural Change

Panel (a) shows log GDP relative to 1980; panel (b) shows the services share of employment. Benchmark denotes continuation, Exposure the infectious-exposure scenario, and WFH the work-from-home scenario. The three paths share the capital and skill continuation and match GDP in 2030; output need not coincide thereafter.



(a) Routine Employment Share

(b) Managerial Employment Share

Fig. 22: Infectious Exposure and Work-From-Home Counterfactuals

Panels show employment shares as fractions of total employment. The scenario labels and common capital, skill and 2030 GDP conditions are the same as in Figure 21. Occupational employment shares are free to differ even when GDP is matched.

7 Conclusion

Employment expanded at both ends of the U.S. occupational wage distribution between 1980 and 2019, but relative wage gains were concentrated at the top and were largest among managers. We have developed a framework that connects these different movements to task productivity, skill requirements, and the distribution of worker and managerial skills. A parsimonious additive requirement links comparative advantage to the skills used in each task, with explicit skill derivatives and a threshold representation of occupational assignment. This makes the same specification useful for analytical comparisons and quantitative exercises. Selection between workers and managers links the rewards to skills with occupational allocation. The analytical results distinguish wage effects among workers from those between workers and managers under explicit sufficient conditions. In the quantitative model, manual employment can expand without the direct skill-return channel operating in nonmanual work, while managerial earnings depend on a separate skill and selection margin.

The counterfactuals at the calibrated parameters show that task-productivity changes account for much of employment reallocation, but produce too little abstract and managerial relative-wage growth and too much managerial employment growth. Changes in requirements and skill supply help reconcile the long-run wage pattern that task productivity alone misses. Both channels can reflect a common technological change. The occupational task evidence supports this interpretation: routine-manual content is associated with the recovered productivity changes, while analytical content and educational attainment are associated with changes in effective skill. These associations provide descriptive empirical support for upskilling, but do not establish it as the unique cause.

The applications illustrate why this distinction matters for technologies and policies with different occupational incidence. In both the broad-AI and LLM-exposure illustrations, abstract wages relative to the economy-wide mean grow faster but abstract employment grows more slowly than in the continuation benchmark. The magnitude depends on the exposure pattern and the specified skill changes. Routine employment is higher under both measures, while relative wages fall under broad AI and rise under LLM exposure. These conditional comparisons illustrate how exposure can translate differently into wages and employment. The policy exercises make a related distinction: preserving routine employment has an output cost, and expanding manufacturing recovers only part of the decline in routine work. The WFH scenarios show how a different direction of task-productivity growth reallocates employment even at a common output level. Across these exercises, the returns to skills and occupational selection are central to interpreting the movement of jobs.

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Occupation Group		occ1990dd
Managers	self-employment+	4– 19
Management Support		22– 37
Professionals		43–199
Technicians		203–235
Sales		243–283
Administrative Support		303–389
Low Skill Services		405–472
Mechanics and Construction		503–599
Miners and Precision		614–699
Machine Operators		703–799
Transportation Workers		803–899

Table A.1: Census Occupation Groups

322 non-farm occupations according to `occ1990dd` (Dorn, 2009), itself harmonized from `occ1990` (Meyer and Osborne, 2005), are grouped into 11 occupation groups in order of their `occ1990dd` code. Except for management support, technicians, and sales, these groups correspond to one-digit Census occupation groups. The table lists groups in ascending code order and excludes agricultural occupations 473–498. In the main text, occupation groups are presented in ascending order of skill (mean hourly wage).

Online Appendix

A Employment, Wages, and Occupational Classification

We use Census and ACS samples from IPUMS USA. We drop military, unpaid family workers, and individuals who were in correctional or mental facilities. We also drop workers who work either in an agricultural occupation or industry.

For each individual, (annual) employment is defined as the product of weeks worked times usual weekly hours, weighted by census sampling weights. Missing usual weekly hours are imputed by hours worked last week when possible. Missing observations are imputed from workers in the same year-occupation-education cell with 322 occupations×6 hierarchical education categories: less than high school, some high school, high school, some college, college, and more than college.

Hourly wages are computed as annual wage-and-salary income divided by annual hours worked at the individual level. Employment shares include the self-employed, while wage means exclude self-employed observations and self-employment income. For the eleven model occupation groups, we first reassign self-employed individuals to management and compute labor-supply-weighted employee wage means within detailed occupations. We then aggregate these detailed means using each occupation's total labor supply, including self-employed individuals, as weights. The resulting broad-group wage means and wage-bill shares therefore use the employment weights of the model groups, rather than employee-only weights at both stages. The manufacturing wage-bill share is constructed analogously using detailed manufacturing wage means and total manufacturing labor supply.

We correct for top-coded incomes by multiplying them by 1.5, and hourly wages are set to not exceed this value divided by 50 weeks × 35 hours (full-time, full-year work). Low incomes are bottom-coded to the first percentile of each year's wage distribution.

For the line graphs in Figures 1–3(b), we ranked occupations by their hourly wages defined as above, and smoothed across skill percentiles using a bandwidth of 0.75 for employment and 0.4 for wages; these are the same values used in Autor and Dorn (2013). For the bar graphs in Figures 1–3(b), 17 and F.8, we group the 322 occupations broadly by their 1-digit Census Occupation Codes, yielding the 11 categories

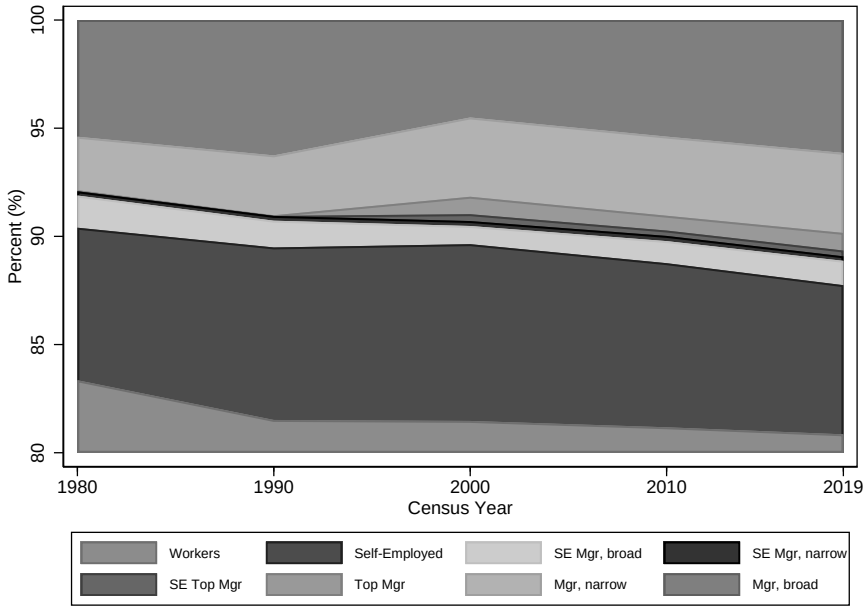


Fig. A.1: Managers in the Census

Source: U.S. Census and ACS. Top managers are coded 4 in `occ1990dd` while broad managers include code 22 which are not-elsewhere-classified managers, or managerial occupations not consistently identified across the sample years.

summarized in Table A.1 and used for our quantitative analysis. In the figures and in Table F.1, these groups are then ranked by the mean wage of the entire group. In particular, in Figures 1–3(b), and F.8, the horizontal length of a bar is set to equal the corresponding group's 1980 employment share, which does not necessarily coincide with the 3-digit occupations used to generate the smooth graphs by percentile.

Our benchmark employment classification includes the self-employed in the manager group. Figure A.1 separates the managerial and self-employed categories and shows that the employment share of non-managerial self-employed workers is comparatively stable. The benchmark includes the three self-employed categories, top managers and narrow managers, but excludes broad managers. Top managers are coded 4 in `occ1990dd` and include CEOs, public administrators and legislators. Broad managers are coded 22 and are either not elsewhere classified or unavailable in some censuses.

B Definition of Competitive Equilibrium

All markets are perfectly competitive. Individuals only care about which task they perform and are indifferent across sectors, and choose the task that yields the highest wage. Let $w_h(h)$ denote the wage of a worker with human capital h, and let w_z denote the efficiency wage per unit of managerial skill z for managers—the latter is possible since all managers perform the same task and we assume a constant returns to scale technology. Then an individual with skill $s = (h, z)$ can choose management if

$$w_z z \geq w_h(h). \tag{38}$$

At equality either occupation is optimal, and a positive mass of tied types can be split across occupations. As in the planner's problem, the density notation below describes the regular case; general type allocations

are measures adding up to μ.

The final good is produced by a representative firm that takes sectoral prices as given, whose profits are given by

$$PY - p_m Y_m - p_s Y_s,$$

where Y is given in (4), P is the final good price and p_i the price of the sector $i \in \{m, s\}$ good. Normalizing $P = 1$, profit maximization yields (27) in the main text.

A manager with skill z in sector i maximizes profits taking the rental rate of capital R and worker wages $w_h(h)$ as given. We exploit constant returns to scale to aggregate over managers in (1).

As in the planner's problem, let $l_{iz}(s)$ denote the number of individuals with skill $s = (h, z)$ working as managers in sector i, then $Z_i \equiv \int z l_{iz}(s) ds$ is the total amount of manager skill in sector i. To this end, define

$$[l_{iz}(s), l_{ih}(s, j), K_{iz}, K_{ih}(j), Z_i, H_i(j), X_{iz}, X_{ih}, T_i(j)]$$

identically as in the main text, except now they are equilibrium allocations to solve for rather than the solution to the planner's problem. In what follows, we solve the problem of a representative sector-i firm that uses technology (7).

Let c_z and $c_h(j)$ denote the unit cost of producing 1 unit of task z and task j output, respectively. Then the sector i firm's problem is

$$\begin{aligned}
&\max \left\{ p_i Y_i - c_z X_{iz} - \int_0^J c_h(j) T_i(j) dj \right\}, \quad \text{where} \\
&c_z = \min \{ R K_{iz} + w_z Z_i \} \quad \text{s.t.} \quad X_{iz} = 1, \\
&c_h(j) = \min \left\{ R K_{ih}(j) + \int w_h(h) l_{ih}(s, j) ds \right\} \quad \text{s.t.} \quad T_i(j) = 1.
\end{aligned}$$

In equilibrium, all firms maximize profits and capital and labor markets clear. Profit maximization implies

$$c_z = p_i A_i^{1-1/\omega} \left[\frac{\eta_i Y_i}{X_{iz}} \right]^{\frac{1}{\omega}}, \quad c_h(j) = p_i A_i^{1-1/\omega} \left[\frac{(1-\eta_i) Y_i}{X_{ih}} \right]^{\frac{1}{\omega}} \left[\frac{\nu_i(j) X_{ih}}{T_i(j)} \right]^{\frac{1}{\sigma}} \tag{39a}$$

and cost minimization implies that the amount of capital chosen for each task, and total units of managerial skill chosen for the managerial task, satisfy

$$\alpha c_z X_{iz} = R K_{iz}, \quad \alpha c_h(j) T_i(j) = R K_{ih}(j), \quad (1-\alpha) c_z X_{iz} = w_z Z_i. \tag{40a}$$

Similarly, the optimal choice of labor satisfies

$$(1-\alpha) c_h(j) T_i(j) \cdot b(h, j) \leq w_h(h) H_i(j) \quad \text{with equality if } l_{ih}(s, j) > 0. \tag{40b}$$

We already imposed market clearing of the sectoral goods in (7) and (27), that is, we used Y_i to denote both the supply and demand for sectoral goods. The equilibrium capital and labor market clearing conditions are identical to the resource constraints (5) and (6), respectively, where K is the exogenously given supply of capital and K_i are the sectoral supply and demand.

Definition 1 *A competitive equilibrium is a set of capital allocations* $\{K_{iz}, K_{ih}(j)\}$*, labor allocations* $\{l_{iz}(s), l_{ih}(s,j)\}$*, and prices* $\{p_i, R, w_z, w_h(h)\}$ *satisfying* (5)-(6)*,* (27)*, and* (40) *for* $i \in \{m, s\}$*, with each individual choosing an earnings-maximizing activity. Management is optimal under* (38)*, and tied types may be split. Labor allocations and clearing are understood as measures for general skill distributions and as the displayed densities in the regular case.*

Substituting the task costs c_z and $c_h(j)$ into (40) recovers the allocation conditions derived from the planner's problem. The planner formulation suppresses these intermediate prices and simplifies the notation.

C Equilibrium Proofs

C.1 Proof of Lemma 1 and Corollary 1

The feasibility constraint (6) and existence of $\hat{h}(j)$ and $\hat{z}(j)$ imply that the number of people with skill s assigned to task j is

$$l_h(s,j)ds = \delta(j - \hat{j}(h)) \cdot \mathcal{I}\left[z \leq \tilde{z}(h)\right] d\mu$$

where $\delta(\cdot)$ is the Dirac delta function and $\mathcal{I}$ the indicator function. Hence the allocation rule is completely determined by the assignment functions $\hat{h}(j)$ and $\hat{z}(j)$, and the productivity of all workers assigned to task $j = \hat{j}(h)$ is

$$H(j) = \int b(h', \hat{j}(h')) \cdot \delta(j - \hat{j}(h')) \cdot F(\tilde{z}(h')|h') dG(h').$$

With the change of variables $j' = \hat{j}(h')$, we can instead integrate over j':

$$\begin{aligned} H(j) &= \int b(\hat{h}(j'), j') \cdot \delta(j - j') \cdot F(\hat{z}(j')|\hat{h}(j')) g(\hat{h}(j')) \cdot \hat{h}'(j') dj' \\ &= b(\hat{h}(j), j) \cdot F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j)) \cdot \hat{h}'(j), \end{aligned}$$

which is (17).

For the optimal allocation, there can be no marginal gain from switching any worker's assignment. So for any $j' = j + dj$,

$$\begin{aligned} \frac{MPT_i(j) \cdot T_i(j)}{H_i(j)} \cdot b(\hat{h}(j), j) &\geq \frac{MPT_i(j') \cdot T_i(j')}{H_i(j')} \cdot b(\hat{h}(j), j'), \\ \frac{MPT_i(j') \cdot T_i(j')}{H_i(j')} \cdot b(\hat{h}(j'), j') &\geq \frac{MPT_i(j) \cdot T_i(j)}{H_i(j)} \cdot b(\hat{h}(j'), j), \end{aligned}$$

with equality in the limit as $|dj| \to 0$, and where $MPT_i(j)$ is the marginal product of X_{ih} with respect to $T_i(j)$. Using (9) to substitute out $MPT_i(j) \cdot T_i(j)$ ratios, and substituting for $H_i(j) = q_{ih}(j)H(j)$ using (17), we obtain

$$\frac{b(\hat{h}(j'), j')}{b(\hat{h}(j), j')} \geq \frac{\pi_{ih}(j')}{\pi_{ih}(j)} \cdot \frac{q_{ih}(j)F(\hat{z}(j)|\hat{h}(j))g(\hat{h}(j))\hat{h}'(j)}{q_{ih}(j')F(\hat{z}(j')|\hat{h}(j'))g(\hat{h}(j'))\hat{h}'(j')} \geq \frac{b(\hat{h}(j'), j)}{b(\hat{h}(j), j)},$$

and as $|dj| \to 0$,

$$\left[\partial \log b(\hat{h}(j), j) \big/ \partial h\right] \cdot \hat{h}'(j) = d \log \left\{ \pi_{ih}(j) \Big/ \left[q_{ih}(j) F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j)) \hat{h}'(j) \right] \right\} \Big/ dj.$$

Now using the total derivative of $b(\hat{h}(j), j)$:

$$d \log b(\hat{h}(j), j) \big/ dj = \left[\partial \log b(\hat{h}(j), j) \big/ \partial h\right] \cdot \hat{h}'(j) + \partial \log b(\hat{h}(j), j) \big/ \partial j, \tag{41}$$

and applying $\pi_{ih}(0) = 1$, we obtain (18):

$$H_i(j) \big/ \pi_{ih}(j) H_i(0) = \exp\left[\int_0^j \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj' \right] \equiv B_j(j; \hat{h}). \tag{42}$$

Plugging (13) and (18) into (12) yields the first equality in (20) in the corollary, and note that (41) implies that $b(h, \hat{j}(h)) = B_h(h; \hat{j}) \cdot B_j(\hat{j}(h); \hat{h})$, which yields the second equality.

C.2 Proof of Proposition 1 and Corollary 2

First, we re-express all capital input ratios only in terms of the thresholds $[\hat{h}(j), \hat{z}]$. Plugging (18) into (9), and applying the task production function (8) we obtain

$$\pi_{ih}(j) = v_{ih}(j) \Big/ \left[\tilde{M}(j) B_j(j; \hat{h})^{1-\alpha} \right]^{1-\sigma}, \quad \text{where } v_{ih}(j) \equiv \frac{\nu_i(j)}{\nu_i(0)} \tag{43}$$

and $\tilde{M}(j) \equiv M(j)/M(0)$. Similarly, plugging (10) and (13) in (11) we obtain

$$\pi_{iz} = v_{iz} \cdot \left(\tilde{M}(z) \cdot \hat{z}^{1-\alpha} \right)^{\omega - 1} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{\sigma-1}}, \quad \text{where } v_{iz} \equiv \frac{\eta_i \nu_i(0)^{\frac{1-\omega}{\sigma-1}}}{1-\eta_i} \tag{44}$$

and $\tilde{M}(z) \equiv M(z)/M(0)$.

Now given a between-sector allocation rule $[q_{ih}(j), q_{iz}]$, the optimal within-sector allocation is described by $[\hat{h}(j)]_{j=0}^{J}$ that solves a fixed point defined by (17)-(18) in Lemma 1 and (43), and $\hat{z}$ that solves the fixed point defined by (13) and (44):

$$q_{ih}(j) \hat{h}'(j) = \frac{v_{ih}(j)}{\tilde{M}(j)^{1-\sigma}} \cdot \frac{B_j(j)^{\alpha + \sigma(1-\alpha)}}{b(\hat{h}(j), j)} \cdot \frac{H_i(0)}{F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j))} \tag{45a}$$

$$\hat{z}^{\alpha + \omega(1-\alpha)} = \frac{\tilde{M}(z)^{1-\omega}}{v_{iz}} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{1-\sigma}} \cdot \frac{q_{iz} Z}{H_i(0)} \tag{45b}$$

where the boundary conditions for the ODE in (45a) are $\hat{h}(0) = 0$ and $\hat{h}(J) = h_M$, which determines $H_i(0)$:

$$H_i(0) \cdot \int_0^J \left[\frac{v_{ih}(j) \cdot B_j(j)^{\alpha + \sigma(1-\alpha)}}{q_{ih}(j) \tilde{M}(j)^{1-\sigma} \cdot b(\hat{h}(j), j) \cdot F(\hat{z}(j)|\hat{h}(j)) g(\hat{h}(j))} \right] dj = h_M. \tag{45c}$$

The functions $\left[B_j(j), B_h(h), \hat{z}(j), \tilde{z}(h)\right]$, which represent relative wages in equilibrium, are defined in (16), (19) and (20) and are functions only of $[\hat{h}(j), \hat{z}]$. That is, system (45) is a fixed point only in terms of the thresholds, and unaffected by the total amount of physical capital and labor in either sector. What matters is the relative mass of individuals *across tasks*.

These equations characterize any regular equilibrium. Existence with endogenous sectoral weights follows from Proposition 2; it is not inferred from separate solutions of the boundary-value equation and managerial cutoff equation.

Sectoral production function The corollary expresses sectoral output only in terms of sectoral capital and labor, and the optimal assignment rules. To derive this, first note that using (9)-(11), sectoral capital can be written as $K_i = K_{ih}(0)\Pi_{K_i}$, which is the first equation in (22). Next, from (13), we know that Z_i is linear in $H_i(0)$:

$$Z_i = \underbrace{q_{iz}L_z}_{\equiv L_{iz}}\bar{z} = L_{iz}\bar{z} = H_i(0)\cdot\hat{z}\pi_{iz}, \quad \text{where} \quad L_z \equiv \int_{z>\tilde{z}(h)} d\mu, \quad \bar{z} = Z/L_z,$$

and using Lemma 1 and (43), so is total worker headcount:

$$\begin{aligned}\int \left[H_i(j)\Big/ b(\hat{h}(j),j)\right] dj &= \int q_{ih}(\hat{j}(h))F(\tilde{z}(h)|h)g(h)dh \\ &= H_i(0)\cdot\int \left[\pi_{ih}(j)/B_h(\hat{h}(j))\right] dj \quad = L_i - L_{iz}.\end{aligned}$$

So rearranging, we can represent sectoral labor input as $L_i = H_i(0)\Pi_{L_i}$, which is the second equation in (22). Finally, use (8)-(11) to rewrite (7) as

$$Y_i = \psi_i\cdot\Pi_{K_i}^{\frac{\omega}{\omega-1}}\Pi_{ih}^{\frac{\sigma-\omega}{(\sigma-1)(1-\omega)}}M(0)K_{ih}(0)^{\alpha}H_i(0)^{1-\alpha},$$

and replacing $[K_{ih}(0), H_i(0)]$ with the expressions in (22) yields (21).

C.3 Proof of Lemma 2

The lemma follows from the one-sector case of Proposition 2, proved below, with endogenous occupational choices and prices.

C.4 Proof of Proposition 2

Unit costs. The unit cost of $MK^{\alpha}H^{1-\alpha}$ is $\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}M^{-1}R^{\alpha}p^{1-\alpha}$. Applying the CES unit-cost formula successively to worker tasks, worker and managerial composites, and sectors gives (25). Each transformed elasticity is finite and positive. A positive-weight CES cost is concave, homogeneous of degree one, and strictly concave except along proportional input-price vectors. Applying this property through the nests establishes the corresponding property of C. Each sector has a positive managerial weight, and aggregate task weights are positive almost everywhere. Equality in cost concavity therefore requires proportional worker prices on every sector's used tasks, with the proportionality factor fixed by the common managerial price. Aggregate task coverage forces the same factor throughout the task interval. Distinct continuous worker-price profiles cannot differ only on a cost-irrelevant open set.

Attainment with a continuum of tasks. Let $b_- = \inf_{h,j} r_b(h,j) > 0$, $b_+ = \sup_{h,j} r_b(h,j) < \infty$, and $B_0 = \mathbb{E}b(h,0) \in (0,\infty)$. Uniform continuity gives the task modulus

$$m_b(r) = \sup_{h,\,|j-j'|\le r} |\log r_b(h,j) - \log r_b(h,j')| \longrightarrow 0 \quad \text{as } r \downarrow 0.$$

For any continuous nonnegative worker-price profile let $w_p(h) = \max_j p_h(j)b(h,j)$, and define its maximal supporting profile

$$\bar{p}_h(j) = \inf_{h \in \mathcal{H}} \frac{w_p(h)}{b(h,j)}.$$

Then $\bar{p}_h \geq p_h$ and $w_{\bar{p}}(h) = w_p(h)$ for every h. Replacing p_h by $\bar{p}_h$ leaves $\mathcal{G}$ unchanged and weakly raises C. Whenever the worker profile is nonzero, $\log \bar{p}_h$ has modulus of continuity bounded by m_b, and

$$\frac{b_-}{b_+}\|p_h\|_\infty \leq \bar{p}_h(j) \leq \frac{b_+}{b_-}\|p_h\|_\infty.$$

On the feasible set $\mathcal{G} \leq 1$, worker prices are bounded by $1/(b_- B_0)$ and the managerial price by $1/\mathbb{E}z$. Supporting worker profiles are uniformly bounded and equicontinuous, including sequences tending uniformly to zero. The supporting-profile transformation is continuous in the uniform norm, so its fixed-point class is closed. Arzela–Ascoli and the bounded managerial-price interval give a compact search set. The bound

$$|\mathcal{G}(p) - \mathcal{G}(\tilde{p})| \leq b_+ B_0 \|p_h - \tilde{p}_h\|_\infty + \mathbb{E}z\,|p_z - \tilde{p}_z|$$

establishes continuity of $\mathcal{G}$. The CES cost index is continuous on this compact class: nonzero supporting worker profiles are uniformly comparable, and profiles tending to zero have worker cost indices tending to zero. The outer finite-dimensional CES limits are continuous. Hence (26) attains its maximum.

Positive prices and uniqueness. An all-positive feasible price vector has positive cost, and homogeneity makes $\mathcal{G} = 1$ at the optimum. If $\Omega \geq 1$, a zero managerial price or an identically zero worker-price profile makes every sectoral cost, hence C, zero. Such a vector cannot maximize. If $0 < \Omega < 1$, introducing a missing price component at size $d > 0$ raises the sectoral cost by order $d^{1-\Omega}$; the required rescaling to preserve $\mathcal{G} \leq 1$ costs at most order d. The former dominates for small d. Thus both components are positive. The supporting-profile bound then makes every worker-task price positive.

Suppose two distinct maximizers satisfy $\mathcal{G} = 1$. Convexity of $\mathcal{G}$ makes their midpoint feasible. Strict concavity of C makes its value strictly larger unless the price vectors are proportional. Proportional maximizers with the same earnings normalization coincide. Thus normalized equilibrium prices are unique.

Market clearing and decentralization. The convex constraint has strictly feasible points. At a positive maximizer the first-order condition is

$$\nabla C = \lambda S, \qquad S \in \partial\mathcal{G}, \qquad \lambda = C,$$

where the last equality uses homogeneity and $\mathcal{G} = 1$. The subgradient S is an occupational service supply induced by earnings-maximizing choices, with tied types split when necessary; the cost gradient is conditional service demand. Consequently $S = \nabla C/C$ clears the labor-service markets. Writing C_* for the maximizing cost, restore the final-good numeraire by multiplying all service prices by

$$s_* = (1-\alpha)K^\alpha C_*^{-(1-\alpha)}, \qquad R = \frac{\alpha s_*}{(1-\alpha)K}, \qquad Y = K^\alpha C_*^{-(1-\alpha)}.$$

These values make output unit cost one and satisfy $RK = \alpha Y$ and total labor earnings $(1-\alpha)Y = s_*$. CES conditional demands uniquely determine aggregate sectoral output, capital, and effective service allocations.

Sectoral allocation identities. The following identities describe the regular interior case with positive sectoral task weights. At zero task weights, use the conditional cost demands directly. Unit elasticities are interpreted by limits. Here $q_h(j)$ and q_z are service-sector shares.

$$Q_h(j) \equiv \frac{q_h(j)}{1-q_h(j)} = Q_h(0) \cdot \frac{v_s(j)}{v_m(j)}, \tag{46}$$

so the dependence of the between-sector allocation rule on the within-sector rule can be suppressed into $Q_h(0)$. Likewise, the rule for splitting individuals between managers and workers, (45b), implies

$$Q_z \equiv \frac{q_z}{1-q_z} = Q_h(0) \cdot \frac{v_{sz}}{v_{mz}} \cdot \left(\frac{\Pi_{mh}}{\Pi_{sh}}\right)^{\frac{\sigma-\omega}{1-\sigma}} \tag{47}$$

where Π_{ih} is defined by (10) and (43), and thus depends on the thresholds $[\hat{h}(j), \hat{z}]$ through B_j. So in equilibrium, $Q_h(0)$ alone must solve the implied sectoral shares in (23) given (21):

$$Q_h(0) = \frac{\gamma_s}{\gamma_m}\left(\frac{\psi_s}{\psi_m}\right)^{\epsilon-1}\left(\frac{\Pi_{sh}}{\Pi_{mh}}\right)^{\frac{(\sigma-\omega)(1-\epsilon)}{(1-\sigma)(1-\omega)}}\left(\frac{\Pi_{K_s}}{\Pi_{K_m}}\right)^{\frac{\omega-\epsilon}{1-\omega}}. \tag{48}$$

Here $L_s/L_m = Q_h(0)\Pi_{L_s}/\Pi_{L_m}$ and $\bar{w}_s/\bar{w}_m = (\Pi_{K_s}/\Pi_{K_m})/(\Pi_{L_s}/\Pi_{L_m})$; substituting these ratios into (23) cancels the Π_L terms.

Uniqueness of occupational assignment. Strict log-supermodularity implies that maximizers of $p_h(j)b(h,j)$ are ordered in h. If one h has two distinct maximizing tasks, no smaller h can choose above the lower one and no larger h can choose below the upper one. The open task intervals associated with such ties are disjoint; choosing a rational point in each shows that there are at most countably many tied worker-skill types. The atomless selected worker-skill measure assigns them zero worker mass; atoms among types who only manage do not affect this conclusion. The stated zero-probability condition on $z = w_p(h)/p_z$ removes worker–manager ties; conditional managerial atomlessness suffices for it. Occupational choices are therefore unique almost everywhere.

Under the regular interior characterization, positive demand densities and positive worker participation at each h make the inverse assignment continuous and strictly increasing. Under the additional regularity imposed for differential characterization it satisfies the density equation (17). The service-price uniqueness does not distinguish observationally equivalent individual assignments across sectors or managers. Such assignments leave the same aggregate effective services and output.

D Comparative Statics Proofs

This appendix establishes the local task-automation result, the skill and requirement comparative statics, and sectoral reallocation. After routinization, we prove the relative-productivity result before the requirement result, since the latter uses the same price-response coefficients. In these proofs, $p(j) = p_h(j)$, $p_z = w_z$, and $w(h) = w_h(h)$.

D.1 Proof of Proposition 3

We allow individuals to switch between worker and managerial occupations throughout the proof.

Baseline coefficients and regularity. Normalize $p_z = 1$, write $\ell = \log b_\delta$, $t(h) = w(h)/p_z$, and let $\mathcal{G}$ be total labor earnings. Define

$$\tau(j) = \frac{p(j)H(j)}{\mathcal{G}}, \quad \tau_z = \frac{Z}{\mathcal{G}}, \quad T = \int_0^J \tau = 1 - \tau_z, \quad e(j) = \frac{t(\hat{h}(j))f(t(\hat{h}(j)) \mid \hat{h}(j))}{F(t(\hat{h}(j)) \mid \hat{h}(j))},$$

$$D(j) = \frac{w(\hat{h}(j))F(t(\hat{h}(j)) \mid \hat{h}(j))g(\hat{h}(j))}{\mathcal{G}\ell_{hj}(\hat{h}(j), j)}, \quad c = 1 - \sigma, \quad d = 1 - \omega, \quad \zeta = \Omega + \frac{\Sigma - \Omega}{T}.$$

Here $c, d > 0$, $\Sigma = \alpha + (1-\alpha)\sigma$, and $\Omega = \alpha + (1-\alpha)\omega$. A subscript 0 denotes the limiting baseline as $\delta \downarrow 0$, not the shocked equilibrium. Positivity and smoothness in Assumption 5 apply on the compact worker/task domains and a neighborhood of the selected cutoffs. The total managerial first moment is finite; derivatives of selected moments follow from the density at the moving cutoffs, without a smooth global tail.

At $\delta = 0$, optimality gives $p_0(j)k_0(j) = q_0$ and $w_0(h) = q_0 b_0(h)$. The selected worker wage bill and managerial services are

$$W(q) = \int qb_0(h)F(qb_0(h) \mid h)g(h)dh, \quad Z(q) = \int \int_{(qb_0(h),\infty)} z\, dF(z \mid h)dG(h).$$

The worker cost index is proportional to q, so demand requires $W(q)/Z(q) = Cq^{1-\Omega}$ for a fixed $C > 0$. The logarithmic elasticity of W is at least one, Z is nonincreasing, and the demand elasticity $1 - \Omega$ is below one. This gives a unique interior q_0; existence and positive aggregate supplies follow from the price argument, which does not require strict log-supermodularity. The limiting inverse assignment is the unique cumulative wage-bill match

$$\frac{1}{\mathcal{G}_0}\int_0^{\hat{h}_0(j)} w_0(h)F(w_0(h) \mid h)g(h)dh = \int_0^j \tau_0(s)ds. \tag{49}$$

It is a canonical limit of positive sorting, not a uniqueness claim for individual assignments at the exactly separable technology.

For small positive δ, task optimality implies

$$p_\delta(j)k_0(j) = p_\delta(0)k_0(0)\exp\left\{-\delta\int_0^j u_j(\hat{h}_\delta(s), s)ds\right\}.$$

The bounded integrand makes the relative price spread $O(\delta)$. The strictly monotone scalar clearing equation above bounds its level and gives $q_\delta \to q_0$. Hence wages, selected supplies, and demand densities converge uniformly. Their positive density bounds and cumulative clearing give $\hat{h}_\delta \to \hat{h}_0$ in C^1. In particular, on each smooth task interval,

$$\tau_\delta \to \tau_0, \quad e_\delta \to e_0, \quad \delta D_\delta \to \bar{D}_0(j) = \frac{w_0(\hat{h}_0(j))F(w_0(\hat{h}_0(j)) \mid \hat{h}_0(j))g(\hat{h}_0(j))}{\mathcal{G}_0 u_{hj}(\hat{h}_0(j), j)} > 0. \tag{50}$$

The smoothness and uniform positive bounds make these limits uniform, with the corresponding first

derivatives. Assumption 6 requires only that e_0 be nondecreasing along the reference selection curve. No sign restriction on e'_δ away from that limit, constant selection elasticity, or exact power conditional c.d.f. is required.

Exact local response. Let $R = (\underline{j}, \overline{j})$, $r = T^{-1}\int_R \tau$, and $x(j) = \partial_m \log[p(j)/p_z]$, evaluated at $m = 0$ for fixed $\delta > 0$. The envelope theorem and task optimality give

$$\partial_m \log[w(h)/p_z] = x(\hat{j}(h)), \qquad \partial_m \hat{h}(j) = -\frac{x'(j)}{\ell_{hj}(\hat{h}(j), j)}. \tag{51}$$

Set $V = Dx'$ and $I = \int \tau x$. Linearized supply and CES demand clearing give

$$-V' + \tau(e + \Sigma)x = \tau\{B - c\mathbf{1}_R\}, \qquad B = \zeta I + (c - d\tau_z)r, \qquad V(0) = V(J) = 0. \tag{52}$$

Both x and V are continuous at the shock boundaries. The one-sided derivatives obey $V'(\underline{j}+) - V'(\underline{j}-) = c\tau(\underline{j})$ and $V'(\overline{j}+) - V'(\overline{j}-) = -c\tau(\overline{j})$. For completeness, supply-share growth is $\tau\{(1 + e)x - I\} - V'$. Within-worker demand-share growth is $(1 - \Sigma)(x - I/T) - c(\mathbf{1}_R - r)$; total worker-share growth is $-(1 - \Omega)\tau_z\{I/T - cr/(1 - \Sigma)\}$. Equating supply and demand and using $(1 - \Omega)c/(1 - \Sigma) = d$ gives (52). This includes the managerial clearing row through adding up.

Equivalently the response operator is

$$\begin{aligned}\mathcal{A}x = & -(Dx')' + \tau(e + \Omega\tau_z)x + \zeta\tau(j)\int \tau(s)[x(j) - x(s)]ds \\ & = \tau\{(c - d\tau_z)r - c\mathbf{1}_R\}.\end{aligned} \tag{53}$$

Its quadratic form, with $\bar{x} = T^{-1}\int \tau x$, can be written

$$\int D(x')^2 + \int \tau e x^2 + \Sigma \int \tau(x - \bar{x})^2 + \Omega\tau_z T\bar{x}^2 > 0$$

for every nonzero x. This identity uses $\Sigma - \zeta T = \Omega\tau_z$ and remains coercive when $\zeta < 0$; the ordering $\omega < \sigma$ is unnecessary. The positive bounds on τ_z, Σ and Ω give a uniform weighted L^2 bound on x. Since D is bounded below by a positive constant times $1/\delta$, the energy identity also bounds $\|x'\|_2$ by $O(\sqrt{\delta})$. The one-dimensional bound on oscillations gives a uniform $\|x\|_\infty$ bound. Equation (52) then bounds V' and V, while (50) implies $x' = O(\delta)$.

The local differentiability used here can be formulated in piecewise smooth spaces with fixed breakpoints $\underline{j}, \overline{j}$. Write $y = \log p$ and recover $h_y(j)$ from $y'(j) = -\ell_j(h_y(j), j)$, using $\ell_{hj} > 0$. Set $w_y(h_y(j)) = e^{y(j)}b(h_y(j), j)$, and compute $W[y]$, $Z[y]$ and $\mathcal{G}[y] = W[y] + Z[y]$ from the selected supplies. Let $\tau_d[y, m]$ be the CES demand wage-bill density. The nonlinear clearing residual is

$$\mathcal{R}[y, m](j) = \frac{w_y(h_y(j))F(w_y(h_y(j)) \mid h_y(j))g(h_y(j))}{\mathcal{G}[y]}h'_y(j) - \tau_d[y, m](j).$$

The boundary conditions are $y'(0) = -\ell_j(0, 0)$ and $y'(J) = -\ell_j(h_M, J)$, with y, y' continuous at the two breakpoints. A zero of this residual clears workers task by task; managerial clearing follows by adding up. For fixed $\delta > 0$, this is a smooth map from piecewise C^2 prices with these boundary and matching conditions to piecewise C^0 residuals, near the interior equilibrium with $h'_y > 0$. Its derivative, up to an overall nonzero sign, is the operator in (53). Coercivity, one-dimensional elliptic regularity and

the finite-rank integral term give its bounded inverse. The implicit function theorem supplies a locally differentiable piecewise C^2 price branch and piecewise C^1 assignment branch. Finite difference quotients converge in those spaces. This argument is local around the positive interior equilibrium; it makes no statement about a nonregular boundary allocation.

Limiting wages and flux. Integrating (52), using $V(0) = V(J) = 0$ and $\Sigma - \zeta T = \Omega\tau_z$, gives

$$\int \tau e x + \Omega\tau_z I = -d\tau_z T r.$$

Consequently x_δ converges uniformly to the negative constant

$$x_0 = -\frac{d\tau_{z0} r_0}{\bar{e}_0 + \Omega\tau_{z0}} < 0, \qquad \bar{e}_0 = T_0^{-1} \int \tau_0 e_0. \tag{54}$$

Substituting this into (52) gives a continuous limiting flux, with piecewise C^1 convergence,

$$V_0'(j) = \tau_0(j)\{x_0[e_0(j) - \bar{e}_0] + c[\mathbf{1}_R(j) - r_0]\}, \qquad V_0(0) = V_0(J) = 0. \tag{55}$$

Define $H(j) = x_0[e_0(j) - \bar{e}_0] - cr_0$. This function is nonincreasing. Moreover,

$$H(0) < 0 \quad \Longleftrightarrow \quad d\tau_{z0}[\bar{e}_0 - e_0(0)] < c[\bar{e}_0 + \Omega\tau_{z0}]. \tag{56}$$

Assumption 6 imposes this inequality. The simpler check $\tau_{z0} \le c/d$ implies it, including equality in that share bound, since $e_0(0) \ge 0$ and $\Omega\tau_{z0} > 0$. Higher managerial shares are allowed when participation elasticities vary little across worker skills.

The single crossing. Since $H < 0$, integrating (55) from zero gives $V_0 < 0$ on $(0, \underline{j}]$; integrating backward from J gives $V_0 > 0$ on $[\overline{j}, J)$. Within R, in the wage-bill coordinate $s = \int_0^j \tau_0$, the derivative $dV_0/ds = H(j) + c$ is nonincreasing. Thus V_0 is concave there. Its negative left value and positive right value imply exactly one zero $j_0^* \in R$, with strictly positive derivative at that zero. The endpoint derivatives are strictly negative. Uniform convergence and piecewise C^1 convergence preserve these endpoint signs, the transverse interior crossing, and strict signs away from those three zeros. Hence, for all sufficiently small positive δ, V_δ has exactly one interior zero $j_\delta^* \in R$, is negative below it and positive above it.

By (51), the inverse-assignment response has the opposite sign because $D\ell_{hj} > 0$. It is positive below j_δ^* and negative above it. It vanishes at the fixed endpoints with nonzero inward slopes. Piecewise C^1 differentiability of the equilibrium branch preserves these signs and the unique transverse crossing for each fixed sufficiently small δ and every sufficiently small positive m. This proves part 1; the allowed shock size may depend on δ.

Wages and occupational entry. Uniform convergence to the strictly negative constant (54) gives $x_\delta(j) < 0$ everywhere for small positive δ. The cutoff $t(h)$ therefore falls at every fixed h, and every fixed managerial type gains relative to every fixed worker type. In particular $\hat{z} = t(0)$ falls. Locally,

$$\partial_m L_z = -\int t(h) f(t(h) \mid h) g(h) x(\hat{j}(h)) dh \ge 0, \qquad \partial_m \tau_z = -\int \tau e x - \tau_z \int \tau x > 0.$$

For $h^* = \hat{h}(j^*_\delta)$,

$$\frac{d}{dh}\,\partial_m \log[w(h)/p_z] = \frac{x'(\hat{j}(h))}{\hat{h}'(\hat{j}(h))}$$

is negative below h^* and positive above it. Worker wage growth is therefore locally U-shaped. The finite-shock envelope identity in the text gives the corresponding relative wage comparisons. Managerial entry is strict when the selection density is positive on a set of positive worker-skill measure. These fixed-type results do not sign average managerial earnings: the wage-bill and headcount effects in (36) must still be compared. Nor does the inverse-assignment crossing alone establish headcount or capital-share changes for every occupation, given endogenous entry into management.

D.2 Proof of Proposition 5

We work with a one-sector equilibrium. Let $p(j) = p_h(j)$, $p_z = w_z$, $w(h) = \max_j p(j)b(h, j)$, and $t(h) = w(h)/p_z$. Write $\mathcal{G}$ for total labor earnings and define the wage-bill density and managerial share by

$$\tau(j) = \frac{p(j)H(j)}{\mathcal{G}}, \qquad \tau_z = \frac{p_z Z}{\mathcal{G}}, \qquad \tau_W = \int \tau(j)dj = 1 - \tau_z.$$

Use Σ, Ω from (25), and put

$$e(j) = \frac{t(\hat{h}(j))f(t(\hat{h}(j)) \mid \hat{h}(j))}{F(t(\hat{h}(j)) \mid \hat{h}(j))}, \quad D(j) = \frac{w(\hat{h}(j))F(t(\hat{h}(j)) \mid \hat{h}(j))g(\hat{h}(j))}{\mathcal{G}\,\partial_{hj} \log b(\hat{h}(j), j)}, \quad \zeta = \Omega + \frac{\Sigma - \Omega}{\tau_W}.$$

All derivatives below are local and evaluated at the baseline. Set $a = \log[M_h/M(z)]$ and hold M_h fixed, so $\partial_a \log M(z) = -1$. Normalize $\partial_a \log p_z = 0$ and define the managerial-premium response

$$x(j) = -\partial_a \log p(j).$$

The worker envelope gives $\partial_a \log[w(h)/p_z] = -x(\hat{j}(h))$ for fixed h. This normalization removes a common price-growth component and does not affect relative earnings.

Continuous price response. Differentiating task optimality, $(\log p)'(j) + \partial_j \log b(\hat{h}(j), j) = 0$, gives

$$\partial_a \hat{h}(j) = \frac{x'(j)}{\partial_{hj} \log b(\hat{h}(j), j)}. \tag{57}$$

The fixed-skill task response is $-x'/(\partial_{hj} \log b\, \hat{h}')$ and its earnings-weighted flux is $-Dx'$. Expressing linearized market clearing in terms of the premium x gives

$$-(Dx')' + \tau(e + \Sigma)x = \tau B_M, \quad B_M = (1 - \omega)\tau_z + \zeta \int \tau x, \quad x'(0) = x'(J) = 0. \tag{58}$$

The source $(1-\omega)\tau\tau_z$ reflects the fall in managerial productivity relative to workers. The Neumann conditions follow from fixed endpoint inverse assignments. Equivalently, using $\Omega\tau_z + \zeta\tau_W = \Sigma$,

$$-(Dx')' + \tau(e + \Omega\tau_z)x + \zeta\tau \int \tau(k)[x(j) - x(k)]dk = (1-\omega)\tau\tau_z > 0.$$

This includes worker reassignment, occupational switching, and CES demand responses. The managerial clearing row is redundant by adding up wage-bill shares.

Managers relative to workers. Let $\mathcal{L}q = -(Dq')' + \tau(e + \Sigma)q = \tau$ with Neumann boundary conditions. Its positive zero-order coefficient gives a unique $q > 0$. Put $Q = \int \tau q$. Integrating gives $\int \tau(e + \Sigma)q = \tau_W$, so $Q \leq \tau_W/\Sigma$. Equation (58) therefore has the unique solution

$$x = B_M q, \qquad B_M = \frac{(1-\omega)\tau_z}{1 - \zeta Q} > 0.$$

If $\zeta \leq 0$ the denominator is at least one; if $\zeta > 0$, it is at least $1 - \zeta\tau_W/\Sigma = \Omega\tau_z/\Sigma > 0$. Thus no ordering of ω and σ is needed. Every fixed managerial type gains relative to every fixed worker type, and $\partial_a \log t(h) = -x(\hat{j}(h)) < 0$. Consequently

$$\partial_a L_z = \int t(h) f(t(h) \mid h) g(h) x(\hat{j}(h)) dh \geq 0, \qquad \partial_a \tau_z = \int \tau e x + \tau_z \int \tau x > 0.$$

The employment inequality is strict exactly when the selection density is positive on a set of positive worker-skill measure.

Wage differences among workers. Assumption 8 makes e nondecreasing across tasks along the increasing baseline assignment. A rising segment of x would require a local minimum at j_0 followed by a strictly larger local maximum at $j_1 > j_0$, allowing endpoints with their Neumann conditions. Equation (58) gives

$$[e(j_0) + \Sigma]x(j_0) \geq B_M \geq [e(j_1) + \Sigma]x(j_1),$$

a contradiction. Thus $x' \leq 0$, $\partial_a \hat{h} \leq 0$, and $\partial_a \hat{j} \geq 0$. For $h_2 > h_1$,

$$\partial_a \log \frac{w(h_2)}{w(h_1)} = x(\hat{j}(h_1)) - x(\hat{j}(h_2)) \geq 0.$$

For $V = Dx'$, differentiating (58) after division by τ gives

$$-\left(\frac{V'}{\tau}\right)' + \frac{e + \Sigma}{D} V = -e'x, \qquad V(0) = V(J) = 0.$$

If e is nonconstant and nondecreasing, the strong maximum principle gives $V < 0$ in the interior, establishing the strict version.

If $e = k$ is constant, the unique response is $x = (1-\omega)\tau_z/(k + \Omega\tau_z)$, so worker wage growth is constant and task assignments do not change. Linearity of the local response gives the reversed signs in Corollary 3.

Exact abundance equivalence. For any allocation of the same people and capital, multiplying managerial skills by s replaces Z by sZ and

$$M(z)K_z^{\alpha}(sZ)^{1-\alpha} = M(z)s^{1-\alpha}K_z^{\alpha}Z^{1-\alpha}.$$

The production possibilities and individual earnings therefore coincide with those of the productivity experiment. If p_z^A is the price per unit of scaled skill and p_z^P the equivalent productivity-economy price, $sp_z^A = p_z^P$. In particular, the cutoff in original z units is $w^A/(sp_z^A) = w^P/p_z^P$; the cutoff in scaled units is s times this value. The equivalence in (35) uses original units. A common multiplier on all worker-task productivities can likewise be factored out of every production technology, leaving the relative parameter $M(z)/M_h$. This establishes the relative-productivity parameterization. A uniform multiplication of b itself by s_h is equivalent to $M_h \mapsto M_h s_h^{1-\alpha}$; scaling the argument h is a different experiment unless b is homogeneous in h.

D.3 Proof of Proposition 4 and Extensions

Requirement-shock equilibrium. Retain the baseline coefficients above and put $b = h - \chi(j)$. In this proof, a denotes the requirement perturbation ε. For $\chi_a = \chi + ad$, now let $x = \partial_a \log p$ with $\partial_a \log p_z = 0$, and define

$$v(j) = \frac{d(j)}{b(\hat{h}(j), j)}, \quad A(j) = \frac{d(j)}{\chi'(j)}, \quad y(j) = x(j) - v(j), \quad I_y = \int \tau y, \quad I_v = \int \tau v.$$

The task-indexed function $y(j)$ in this proof is $y_J(j)$ in the statement: it evaluates fixed-h wage growth on the baseline assignment. It does not follow a skill type changing with a. With $k_b(h, j) = \partial_a \log b_a(h, j)|_0 = -d(j)/b(h, j)$, differentiating task optimality gives the task flux

$$Q = D\{x' + \partial_j k_b(\hat{h}(j), j)\} = Dy' - \tau A, \qquad Q(0) = Q(J) = 0. \tag{59}$$

Worker supply shares change by $\tau(y - I_y + ey) - Q'$, while demand shares change by $\tau(x - \int \tau x) - \Omega\tau\tau_z x - \zeta\tau \int \tau(k)[x(j) - x(k)]dk$. The baseline demand identity

$$\frac{\tau'}{\tau} = A_0' + (1 - \Sigma)\frac{\chi'}{b}$$

therefore yields the response equation, including reassignment

$$\begin{aligned} -(Dy')' + \tau(e + \Sigma)y &= \tau\{B_R - A' - AA_0'\}, \\ B_R = \zeta I_y + (\zeta - 1)I_v, &\qquad Dy'|_{0,J} = \tau A|_{0,J}. \end{aligned} \tag{60}$$

The term ey captures managerial selection.

Increasing relative worker wage growth. First put $B_A = e^{A_0}A$ and $S_A = -[A' + AA_0']' \geq 0$. Then

$$-B_A'' + A_0'B_A' = e^{A_0}S_A \geq 0, \qquad B_A(0), B_A(J) \geq 0.$$

The one-dimensional maximum principle gives $A \geq 0$. Since $A \not\equiv 0$, either the forcing or a boundary value is nonzero, and the strong maximum principle gives $A > 0$ in the interior. In particular $I_v = \int \tau\chi' A/b > 0$.

Under Assumption 7, $e = k$ along the baseline selection curve. Set $V = Dy'$. Divide (60) by τ and differentiate to eliminate B_R:

$$-\left(\frac{V'}{\tau}\right)' + \frac{k+\Sigma}{D}V = -A'' - A'A_0' - AA_0'' \geq 0,$$
$$V(0) = \tau(0)A(0) \geq 0, \qquad V(J) = \tau(J)A(J) \geq 0. \tag{61}$$

The forcing and boundary values cannot all vanish, since that would imply $A \equiv 0$. The strong maximum principle for this second-order boundary-value problem therefore gives $V > 0$ on $(0, J)$. Hence

$$\partial_a \partial_h \log \frac{w_a(h)}{p_{z,a}} = \frac{y'(\hat{j}(h))}{\hat{h}'(\hat{j}(h))} > 0.$$

Integrating (60), the boundary flux cancels $\int (\tau A)'$. Using $\tau'/\tau = A_0' + (1-\Sigma)\chi'/b$ then gives

$$I_y = \frac{(1-\Omega)\tau_z}{k + \Omega\tau_z} I_v. \tag{62}$$

For $0 < \omega < 1$, $I_y > 0$. A strictly increasing y with a positive wage-weighted average satisfies $y(J) > 0$, so a nonempty upper skill interval gains relative to managers. If $y(0) < 0$, continuity and strict monotonicity give a unique zero; otherwise no worker type strictly loses. Moreover $\partial_a Z = -k\mathcal{G}I_y/p_z$, $\partial_a \mathcal{G}/\mathcal{G} = I_y$, and $\partial_a \tau_z = -(k + \tau_z)I_y$, so its sign is opposite to $1 - \omega$. The strict wage-gradient conclusion holds for every positive ω, σ. Finally, $Q/\tau = \partial_a \hat{j}_a(h)$ at $j = \hat{j}(h)$ establishes (34).

A sufficient primitive sorting restriction. For the index stretch $A = j$, suppose additionally that $0 < \sigma < 1$, $\chi(j) = \gamma j - 1$, $\gamma > 0$, $\gamma J < 1$, $M(j) = 1$, $\nu(j) = 1/J$, and $g(h) = CS(h)^k$, with C normalizing the density. If

$$\frac{h_M}{J} \leq \frac{(1-\Sigma)\gamma}{k+1}, \tag{63}$$

then the entire interior assignment shifts upward. Indeed $gF = C\rho t^k$ cancels the conditional scale, and baseline supply and demand imply

$$\frac{\hat{h}''}{\hat{h}'} = \frac{(1-\Sigma)\gamma - (k+1)\hat{h}'}{1 + \hat{h} - \gamma j}.$$

By uniqueness of this smooth ODE, its slope cannot cross $q_* = (1-\Sigma)\gamma/(k+1)$; a slope reaching q_* remains constant there. Its mean is $h_M/J \leq q_*$, so $\hat{h}' \leq q_*$ throughout. For $\mathcal{P}V = -(V'/\tau)' + (k+\Sigma)V/D$,

$$\mathcal{P}(\tau j) = \frac{\gamma}{b^2}[(k+1)j\hat{h}' - (1-\Sigma)(1+\hat{h})] \leq -\frac{(1-\Sigma)\gamma}{b} < 0.$$

Since (61) has zero forcing here, $Q = V - \tau j$ satisfies $\mathcal{P}Q > 0$ with zero endpoints, so $Q > 0$ on the interior. Thus $\partial_a \hat{j}_a(h) > 0$ and $\partial_a \hat{h}_a(j) < 0$ away from endpoints. This restriction on skill dispersion relative to the requirement gradient is sufficient and strong; it is not part of the general model or the wage-gradient proposition.

An occupational-mean comparison under an additional restriction. For this mean-earnings comparison, retain $0 < \omega < 1$ and take the independent, bounded-power case $S(h) = S_0$ and $\rho = 1$. Put

$u = t/S_0$, $A = \mathbb{E}[u^k]$, and $B_1 = \mathbb{E}[u^{k+1}]$. Then

$$\frac{\bar{w}_z}{\bar{w}_h} = \frac{k}{k+1}\frac{A(1-B_1)}{B_1(1-A)}.$$

Let m_L and m_W be worker-employment- and worker-wage-bill-weighted averages of y, respectively. Both w and y increase with h, so $m_L \le m_W$ by nonnegative covariance. Equation (62) gives $m_W > 0$, and

$$\partial_a \log(\bar{w}_z/\bar{w}_h) = \frac{k}{1-A}m_L - \frac{k+1}{1-B_1}m_W < 0.$$

The inequality follows because $(k+1)u^k - ku^{k+1} < 1$ for $0 < u < 1$, hence $k/(1-A) < (k+1)/(1-B_1)$. For the relative managerial-productivity decline in Proposition 5, this independent benchmark has constant negative worker-relative growth $y = -x < 0$. Thus $m_L = m_W < 0$ and the same mean-ratio formula gives a rising managerial mean premium. Corollary 3 reverses that sign. Neither conclusion is imposed on correlated skills or a different upper tail.

Smooth common-support skills and compatibility. Consider

$$f_{U,m}(u) = \frac{m}{\mathrm{B}(k/m, q/m)}u^{k-1}(1+u^m)^{-(k+q)/m}, \qquad u > 0, \quad k > 0, \quad q > 1. \tag{64}$$

Here B is the beta function and $m > 0$ controls the transition between the lower body and upper tail. The distribution has a positive smooth density, common unbounded support, and finite mean. With $S(h) = S_0 e^{\beta h}$,

$$\partial_{hz} \log f(z \mid h) = \frac{\beta(k+q)mu^{m-1}}{S(h)(1+u^m)^2} > 0, \qquad u = z/S(h).$$

The log-density of $\log U$ is strictly concave:

$$L(v) = kv - \frac{k+q}{m}\log(1+e^{mv}) + \text{constant}, \qquad L''(v) < 0.$$

Its reverse hazard is strictly decreasing. To see this directly, write $R(v) = e^{L(v)}/\int_{-\infty}^{v} e^{L(t)}dt$, so $R' = R(L' - R)$. If $L' \le 0$, $R' < 0$. If $L' > 0$, strict concavity bounds the integral below $e^{L(v)}/L'(v)$, again giving $R' < 0$. For $\Phi(v) = \log F_U(e^v)$ and $G(x,h) = \Phi(x - \log S(h))$,

$$G_{xx} = \Phi'' < 0, \qquad G_{xh} = -\beta\Phi'' > 0, \qquad G_{xh}/|G_{xx}| = \beta.$$

Thus $\beta > \sup \partial_h \log b$ establishes both monotone likelihood ratio (MLR) ordering of the conditional densities and the stronger global inequality $G_{xh} > |G_{xx}|\partial_h \log b$, hence the reference participation ordering. These global restrictions are properties of this example, not general equilibrium assumptions.

As $m \to \infty$ the density in (64) tends to $kq/(k+q)$ times u^{k-1} below one and u^{-q-1} above one. Its lower body is $[q/(k+q)]u^k$, while

$$\mathbb{E}U_m = \frac{\mathrm{B}((k+1)/m, (q-1)/m)}{\mathrm{B}(k/m, q/m)} \longrightarrow \frac{kq}{(k+1)(q-1)}.$$

The CDF and density converge in the required differentiable norms on compact subsets of $(0,1)$; truncated managerial first moments converge there as well. For the index stretch $A = j$, take a regular limiting

equilibrium with selection arguments strictly below one. Its linearized price operator has the coercive quadratic form used in (53), with $e = k$, and therefore a bounded inverse on the fixed-endpoint function spaces. Together with the stated coefficient and tail-moment convergence, the implicit-function theorem gives continuous differentiable dependence on the skill distribution. Assignments and shock responses consequently converge differentiably; differentiability in the shock alone would not suffice. No additional nonsingularity assumption is needed. At the limit, the solution V_0 of (61) satisfies $V_0(j) > 0$ for $j > 0$ and $V_0'(0) > 0$ by the boundary point lemma. The continuous extension of $V_0(j)/j$ has a positive minimum d_0. For sufficiently large finite m,

$$\|V_m' - V_0'\|_\infty < d_0/2, \qquad V_m(0) = 0,$$

so $V_m(j) \geq d_0 j/2 > 0$ for all $j > 0$. Positive upper-tail relative growth and the falling managerial wage-bill share persist by continuity. These bounds establish robustness uniformly through the endpoints. The argument does not determine a numerical threshold for m. The strictly decreasing lower-tail elasticity of this family and the scale dominance condition also give nonzero managerial sorting in Proposition 5.

A primitive sufficient bound for the selection region. For completeness, the power-body/Pareto-tail limiting family admits a conservative bound avoiding any assumption on the location of endogenous cutoffs. Suppose $M(j) = 1$, $0 < \omega < 1$, any finite $\sigma > 0$, and $\rho = q/(k+q)$. Let $S_{\min}$ and $S_{\max}$ be the minimum and maximum of the conditional scale $S(h)$. Put

$$b_0 = -\chi(0), \quad b_- = -\max \chi, \quad b_+ = h_M - \min \chi, \quad P = e^{[\chi(J)-\chi(0)]/b_-},$$

$$r_0 = b_0/S_{\max}, \quad r_+ = Pb_+/S_{\min}, \quad u_- = b_0/(r_+ S_{\max}), \quad E_U = \frac{kq}{(k+1)(q-1)},$$

$$B_a = \frac{(1-\rho)q}{(q-1)\rho} P^{1-\Omega} (S_{\max}/b_0)^{1-\Omega}, \quad B_b = \frac{r_+ S_{\max} E_U}{b_0 \rho u_-^k} (P/r_0)^{1-\Omega}.$$

A sufficient restriction on fundamentals is

$$\frac{\eta}{1-\eta} M(z)^{\omega-1} > \max\{B_a, B_b\}. \tag{65}$$

Indeed task optimality gives $p(0) \leq p(j) \leq Pp(0)$ and $C_h \leq Pp(0)$. If any type has $t(h)/S(h) \geq 1$, then $r = p_z/p(0) \leq r_+$. For $r \leq r_0$, all cutoffs lie in the Pareto region: the worker wage bill is at least $p(0)b_0\rho$ and the managerial wage bill is at most $p(0)(1-\rho)qb_0^{1-q} S_{\max}^q r^q/(q-1)$. Their ratio divided by $(r/P)^{1-\Omega}$ is at most B_a. For $r_0 < r \leq r_+$, worker participation is at least ρu_-^k and managerial services at most $S_{\max} E_U$, giving the upper bound B_b. CES demand requires the same ratio to equal $[\eta/(1-\eta)] M(z)^{\omega-1} (p_z/C_h)^{1-\Omega}$, contradicting (65). Hence all selection arguments are below one, with a uniform margin by compactness. This bound is sufficient and conservative.

D.4 Decomposing Polarization

Let $V_i(j)$ for each task $j \in \mathcal{J}$ denote the "unnormalized" labor in job j in sector i, defined as

$$\begin{aligned} V_i(j) &\equiv (1-\eta_i)\nu_i(0) M(0)^{\sigma-1} \cdot \pi_{ih}(j)/B_h(\hat{h}(j)) \\ &= (1-\eta_i)\nu_i(j) \left[M(j) B_j(j)^{1-\alpha}\right]^{\sigma-1} / B_h(\hat{h}(j)) \end{aligned} \tag{66a}$$

where we have substituted for $\pi_{ih}(j)$ using (43). Similarly we can define

$$\begin{aligned} V_i(z) &\equiv (1-\eta_i)\nu_i(0)M(0)^{\sigma-1}\cdot(\hat{z}/\bar{z})\pi_{iz} \\ &= \eta_i M(z)^{\omega-1}\left[\nu_i(0)M(0)^{\sigma-1}\Pi_{ih}\right]^{\frac{\sigma-\omega}{\sigma-1}}\cdot\hat{z}^{\alpha+\omega(1-\alpha)}/\bar{z} \\ &= \eta_i M(z)^{\omega-1}V_{ih}^{\frac{\sigma-\omega}{\sigma-1}}\cdot\hat{z}^{\alpha+\omega(1-\alpha)}/\bar{z} \end{aligned} \tag{66b}$$

$$V_{ih} \equiv \nu_i(0)M(0)^{\sigma-1}\Pi_{ih} = \int\left\{\nu_i(j)\left[M(j)B_j(j)^{1-\alpha}\right]^{\sigma-1}\right\}dj \tag{66c}$$

where V_{ih} is the unnormalized counterpart of Π_{ih} in (10), and to derive $V_i(z)$, we substituted for π_{iz} using (44). These $[V_i(j)]_{j\in\mathcal{J}^z}$ are simply the amount of labor in task j unnormalized by $H_i(0)$, so we know that taking the ratio between any pair yields the labor input ratio between the two tasks. Using these expressions we can define the "unnormalized" labor in sector i:

$$V_i = (1-\eta_i)\nu_i(0)M(0)^{\sigma-1}\Pi_{L_i} = V_{il} + V_i(z), \quad \text{where } V_{il} = \int V_i(j)dj. \tag{67}$$

So the definition of Π_{L_i} in (22) implies that employment in each task across both sectors can be expressed as

$$L(j) = \sum_{i\in\{m,s\}}\frac{L_i(j)}{L_i}\cdot L_i = \sum_{i\in\{m,s\}}\frac{V_i(j)}{V_i}\cdot L_i.$$

As in the proof of Proposition 3 (Appendix D.1), suppose $M(j)$ grows to $M^1(j) = M(j)e^m$, $m>0$, for all $j\in\mathcal{J}^1\equiv(\underline{j},\bar{j})$. Denote the log-derivative of X with respect to m by Δ_X. Then

$$\begin{aligned} \Delta_{L(j)} &= \sum_{i\in\{m,s\}}\frac{L_i(j)}{L(j)}\cdot\left[\Delta_{V_i(j)} - \Delta_{V_i} + \Delta_{L_i}\right] \\ &= \sum_{i\in\{m,s\}}\frac{L_i(j)}{L(j)}\cdot\Bigg\{\underbrace{\Delta_{V_i(j)} - \int_{\mathcal{J}^z}\left[\frac{V_i(j')}{V_i}\cdot\Delta_{V_i(j')}\right]dj'}_{w_{ij}} + \Delta_{L_i}\Bigg\}. \end{aligned} \tag{68}$$

Equation (68) separates within-sector occupational changes from sectoral reallocation. The sectoral comparison in the next subsection concerns the latter and includes the response of sectoral average earnings.

D.5 Proof of Proposition 6

Accounting and part 1. Write W_i for the sectoral labor wage bill and $\bar{w}_i = W_i/L_i$. Common capital shares imply

$$\frac{K_m}{K_s} = \frac{W_m}{W_s} = \frac{p_m Y_m}{p_s Y_s} = \frac{\gamma_m}{\gamma_s}\left(\frac{p_m}{p_s}\right)^{1-\epsilon}, \qquad \frac{L_m}{L_s} = \frac{W_m}{W_s}\frac{\bar{w}_s}{\bar{w}_m}. \tag{69}$$

Normalize the managerial service price to one. All workers earn $w=t$, and managers are those with $z>t$. Let θ be the common initial worker share of sectoral labor wage bills. Put

$$\phi = \frac{t}{\bar{z}}\in(0,1), \qquad T = \phi + (1-\phi)\theta, \qquad f_\theta = \frac{(1-\phi)\theta(1-\theta)}{T}, \qquad d = r_m - r_s > 0.$$

Write a dot for the derivative with respect to a at zero, and $x = \dot{\log} t$. The worker-service cost index has derivative $\dot{\log} C_{ih} = x - r_i/(1-\alpha)$. The sectoral cost index has derivative $\dot{\log} C_i = \theta\{x - r_i/(1-\alpha)\}$. The common rental-rate term cancels from relative output prices, giving

$$\dot{\log}(p_m/p_s) = -\theta d < 0. \tag{70}$$

CES final demand and (69) imply

$$\dot{\log}(Y_m/Y_s) = \epsilon\theta d > 0, \qquad \dot{\log}(W_m/W_s) = -(1-\epsilon)\theta d < 0. \tag{71}$$

Worker wage-bill shares satisfy

$$\dot{\theta}_i = (1-\omega)\theta(1-\theta)\{(1-\alpha)x - r_i\}.$$

Since $L_i = (W_i/w)T_i$, with $T_i = \phi + (1-\phi)\theta_i$, common changes in w and ϕ cancel in the relative derivative at the equal-θ baseline. Hence

$$\boxed{\dot{\log}(L_m/L_s) = -\{(1-\epsilon)\theta + (1-\omega)f_\theta\}d < 0.} \tag{72}$$

The first term is the relative-revenue contraction under between-sector complementarity. When $\omega < 1$, the second is the change toward higher-paid managers, reducing headcounts per unit of wage bill more in manufacturing. If $\omega > 1$, this composition term opposes the decline; the stated baseline inequality ensures that the first term dominates. If $\lambda_m = L_m/(L_m + L_s)$, then $\dot{\lambda}_m = \lambda_m(1-\lambda_m)\dot{\log}(L_m/L_s) < 0$.

The productivity defined by $Y_i = \Phi_i K_i^\alpha L_i^{1-\alpha}$ obeys

$$\dot{\log}(\Phi_m/\Phi_s) = \{\theta + (1-\alpha)(1-\omega)f_\theta\}d > 0. \tag{73}$$

This follows from (69), (71), and (72). Writing $G_\theta = (1-\epsilon)\theta + (1-\omega)f_\theta$, the coefficient equals $\Xi\theta + (1-\alpha)G_\theta > 0$ under the stated headcount condition, including when $\omega > 1$. It includes the change in employment composition.

Part 2. First allow an arbitrary smooth managerial-skill distribution. Write F and f here for the marginal managerial-skill CDF and density, integrating out h at the homogeneous-worker benchmark. Let $Z(t) = \int_t^\infty z\, dF(z)$ and $\bar{z} = Z(t)/[1-F(t)]$. Write

$$u_\phi = \frac{d\log(t/\bar{z})}{d\log t} = 1 + \frac{t^2 f(t)}{Z(t)} - \frac{t f(t)}{1-F(t)}.$$

Only this local selected-mean elasticity enters the additional headcount-composition term.

Let s_i be sectoral revenue shares and θ_i worker shares of sectoral wage bills. Define $E_i = \theta_i r_i$. The formulas below need no ordering of θ_m and θ_s. In the sufficient ordered-weight example, $\eta_m \le \eta_s$ implies $\theta_m \ge \theta_s$ and hence $E_m > E_s$. Let

$$A = \sum_i s_i\theta_i(1-\theta_i), \quad B = s_m s_s(\theta_m - \theta_s)^2, \quad C = \sum_i s_i\theta_i(1-\theta_i)r_i,$$
$$\theta = \sum_i s_i\theta_i, \qquad e = tf(t)/F(t), \qquad \Xi = \alpha + (1-\alpha)\epsilon.$$

Differentiating aggregate worker wage-bill supply and demand gives

$$x = -\frac{(1-\omega)C + (1-\epsilon)s_m s_s(\theta_m - \theta_s)(E_m - E_s)}{\Omega A + \Xi B + \theta e}. \tag{74}$$

Indeed supply's derivative with respect to $\log t$ is $\theta(1-\theta) + \theta e$, demand's is $(1-\Omega)A + (1-\Xi)B$, and $\theta(1-\theta) = A + B$. The fixed-price shock to demand is the negative numerator in (74). This accounts for sectoral reallocation as well as occupational selection.

For any θ_i, put

$$T_i = \phi + (1-\phi)\theta_i, \quad f_i = \frac{(1-\phi)\theta_i(1-\theta_i)}{T_i}, \quad G_i = (1-\epsilon)\theta_i + (1-\omega)f_i.$$

Allowing ϕ to change with the selection cutoff, the exact relative-headcount response is

$$\begin{aligned}\dot{\log(L_m/L_s)} = (1-\alpha)x(G_m - G_s) - (r_m G_m - r_s G_s) \\ + \phi u_\phi x\left[\frac{1-\theta_m}{T_m} - \frac{1-\theta_s}{T_s}\right].\end{aligned} \tag{75}$$

All terms on the right are computed from the baseline. The exact condition for the relative employment decline is therefore

$$\begin{aligned}r_m G_m - r_s G_s > (1-\alpha)x(G_m - G_s) \\ + \phi u_\phi x\left[\frac{1-\theta_m}{T_m} - \frac{1-\theta_s}{T_s}\right],\end{aligned} \tag{76}$$

where x is the explicit baseline expression in (74). This condition permits either sign of u_ϕ and requires no tail shape away from the cutoff.

Relative prices obey

$$P = \dot{\log(p_m/p_s)} = (1-\alpha)(\theta_m - \theta_s)x - (E_m - E_s).$$

Thus the exact condition for the price decline is

$$(1-\alpha)(\theta_m - \theta_s)x < E_m - E_s. \tag{77}$$

Revenue and output signs follow from final CES demand. When the two baseline conditions give $P < 0$ and $L = \dot{\log(L_m/L_s)} < 0$, accounting gives

$$\dot{\log(\Phi_m/\Phi_s)} = -\Xi P - (1-\alpha)L > 0.$$

Thus the stated headcount condition also suffices for the TFP sign; no separate selected-mean restriction is needed.

A simple tail example. If $\Pr(z > u) = \pi(z_0/u)^q$ for $u \geq z_0$, with $q > 1$, $0 < \pi < 1$, and $t > z_0$, then $\bar{z} = qt/(q-1)$, $\phi = (q-1)/q$, and $u_\phi = 0$. For $f(\theta) = (1-\phi)\theta(1-\theta)/[\phi + (1-\phi)\theta]$,

$$f'(\theta) = \frac{\phi}{[\phi + (1-\phi)\theta]^2} - 1 \geq -(1-\phi) = -1/q.$$

Consequently $G'(\theta) \geq (1-\epsilon) - (1-\omega)/q \geq 0$ under the primitive bound in the statement, and it is strictly positive at interior worker shares. Under $\eta_m \leq \eta_s$ and $\omega < 1$, (74) gives $x < 0$ and $E_m > E_s$, hence (77). Also $G_m \geq G_s > 0$, so (76) holds. The restriction $\epsilon \leq \omega$ is sufficient but unnecessarily tight. The primitive bound can also be relaxed whenever the exact baseline condition holds. More generally, under the same ordered-weight complementary-manager configuration, $u_\phi \leq 0$ reinforces the decline when $G_m \geq G_s$, because the final bracket in (75) is nonpositive.

A primitive sufficient condition for tail selection. The tail-selection condition can be ensured from primitives. Set $p_z = 1$ and evaluate worker cost $c_0 = z_0 M_0^{-1/(1-\alpha)}$ at $t = z_0$. If

$$\frac{(1-\eta_s)c_0^{1-\Omega}}{\eta_s M(z)^{\omega-1} + (1-\eta_s)c_0^{1-\Omega}} > \frac{1-\pi}{1-\pi+\pi q/(q-1)}, \qquad \Omega = \alpha + (1-\alpha)\omega,$$

even the more management-intensive sector demands a greater worker wage-bill share than supply at $t = z_0$. Equilibrium therefore has $t > z_0$: supply worker/manager wage-bill odds grow more than proportionally in t, while CES demand odds grow less than proportionally. The inequality is sufficient, not necessary.

Extension to positive worker heterogeneity. At each benchmark all reported signs are strict. For each sufficiently small fixed shock, continuity of equilibrium prices and aggregate allocations therefore preserves these comparisons under sufficiently small perturbations. At $b = 1$, effective worker services equal worker headcounts even though assignments across tasks need not be unique. Under $b_\delta \to 1$ uniformly, that distinction vanishes uniformly; a common managerial mean in the compared allocations likewise identifies managerial headcounts. For local derivative signs, equality of baseline means and their first-order responses is enough. A sufficiently small positive gap in managerial weights is permitted in part 1. Strict log-supermodularity holds for $\delta > 0$. To extend local derivative signs, rather than comparisons for a fixed shock, use the additional continuous-derivative condition in the proposition. Finite-shock continuity alone is insufficient for this stronger conclusion. No quantitative size for these neighborhoods is established.

D.6 Composition Effects

Let $n_{Ri} = L_i(\mathcal{R})/L_i$ be the within-sector routine employment share, $\lambda_m = L_m/(L_m + L_s)$, and n_R the aggregate routine share. Then

$$\dot{n}_R = \lambda_m \dot{n}_{Rm} + (1-\lambda_m)\dot{n}_{Rs} + (n_{Rm} - n_{Rs})\dot{\lambda}_m. \tag{78}$$

Lemma 3 (Composition effect) *If manufacturing's employment share falls and $n_{Rm} > n_{Rs}$, the between-sector term in* (78) *is negative. If services has a higher managerial employment share, the corresponding between-sector contribution to the aggregate managerial share is positive.*

Proof *The first statement follows directly from* (78)*; the second applies the same identity to managers. At the homogeneous-worker benchmarks, $n_{Ri} = \theta_i r_i/T_i$. Since θ_i/T_i increases in θ_i, $r_m > r_s$ and $\theta_m \geq \theta_s$ imply $n_{Rm} > n_{Rs}$. The managerial share is $\phi(1-\theta_i)/T_i$, which decreases in θ_i. It is therefore higher in services in the ordered-weight example with $\eta_s > \eta_m$. At the equal-weight benchmark the managerial composition term is zero. The strict routine-share ordering persists nearby for that example. In the broader*

exact-condition cases, the routine and managerial composition orderings must be checked separately. ∎

This decomposition is evaluated at the full equilibrium response. It does not establish a comparison with an alternative equilibrium holding sector shares fixed.

E Quantitative Model and Numerical Details

With discrete tasks, threshold workers must earn the same wage in adjacent tasks. Write $MPT_{ij} = \partial X_{ih}/\partial T_{ij}$ for the marginal product of task output in the worker composite. Then:

$$MPT_{i0} \cdot \frac{(1-\alpha)T_{i0}}{L_{i0}} = MPT_{i1} \cdot \frac{(1-\alpha)T_{i1}}{(\bar{h}_1 - \chi_1)L_{i1}} \cdot (\hat{h}_1 - \chi_1),$$
$$MPT_{ij} \cdot \frac{(1-\alpha)T_{ij}}{(\bar{h}_j - \chi_j)L_{ij}} \cdot (\hat{h}_{j+1} - \chi_j) = MPT_{i,j+1} \cdot \frac{(1-\alpha)T_{i,j+1}}{(\bar{h}_{j+1} - \chi_{j+1})L_{i,j+1}} \cdot (\hat{h}_{j+1} - \chi_{j+1})$$

where L_{ij} is the measure of workers in sector i, task j, and $\bar{h}_j$ is their mean raw worker skill. For $j \geq 1$, effective worker skill is $H_{ij} = (\bar{h}_j - \chi_j)L_{ij}$; the manual task instead has $H_{i0} = L_{i0}$. We retain the representative within-task allocation with equal mean raw skills across sectors $i \in \{m, s\}$. This equality is automatic when tasks are a continuum. Then

$$\hat{h}_1 - \chi_1 = \frac{(\bar{h}_1 - \chi_1)L_{i1}}{\pi_{i1}L_{i0}}, \qquad \frac{\hat{h}_{j+1} - \chi_{j+1}}{\hat{h}_{j+1} - \chi_j} = \frac{\pi_{ij}(\bar{h}_{j+1} - \chi_{j+1})L_{i,j+1}}{\pi_{i,j+1}(\bar{h}_j - \chi_j)L_{ij}}, \tag{79}$$

where π_{ij} is the discrete version of (9), and can be expressed using (79) as

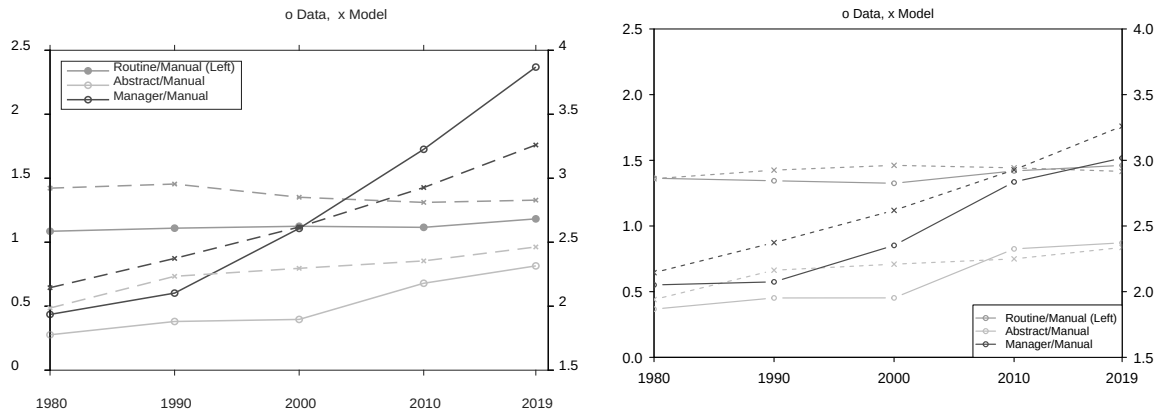
$$\pi_{i1} = \frac{\nu_{i1}}{\nu_{i0}} \cdot \left(\frac{M_1}{M_0} \cdot (\hat{h}_1 - \chi_1)^{1-\alpha}\right)^{\sigma-1}, \quad \frac{\pi_{i,j+1}}{\pi_{ij}} = \frac{\nu_{i,j+1}}{\nu_{ij}} \cdot \left[\frac{M_{j+1}}{M_j}\left(\frac{\hat{h}_{j+1} - \chi_{j+1}}{\hat{h}_{j+1} - \chi_j}\right)^{1-\alpha}\right]^{\sigma-1}. \tag{80}$$

In equilibrium, indifference across tasks for threshold workers implies

$$w_0 = w_z\hat{z} = w_1(\hat{h}_1 - \chi_1), \quad w_j(\hat{h}_{j+1} - \chi_j) = w_{j+1}(\hat{h}_{j+1} - \chi_{j+1})$$
$$\Rightarrow \quad w_z/w_0 = 1/\hat{z}, \quad w_1/w_0 = 1/(\hat{h}_1 - \chi_1), \quad w_{j+1}/w_j = \frac{\hat{h}_{j+1} - \chi_j}{\hat{h}_{j+1} - \chi_{j+1}}. \tag{81}$$

These price ratios are used to select the skill distribution and requirement profile at each endpoint, as described in Section 4.3. Conditional on employment shares, the routine-group mean wage is an employment-weighted average of the seven routine occupation means. That group target therefore summarizes the routine-wage fit rather than supplying an independent wage restriction. The quantitative exercises condition on the endpoint parameter values reported in Tables 5 and 6. The remaining calibration has the following blocks:

1. Choose candidate common elasticities (σ, ω). Their values apply to both the 1980 and 2019 equilibria.
2. Conditional on the 1980 distribution, requirements, and threshold vector $\mathbf{x}_{1980}$, recover the sectoral task weights from 1980 within-sector occupational employment shares. Normalize all initial task



(a) Manufacturing

(b) Services

Fig. F.1: Data vs. Model, Relative Occupational Wages by Sector, 1980–2019.
Routine, abstract, and manager mean wages are divided by the manual mean wage within the same sector and year. Circles denote data and crosses denote the model. The routine ratio uses the left axis; the abstract and manager ratios use the right axis. The broad occupation groups are those in Table 6; these panels complement the aggregate wage fit in Figure 10.

productivities to the common level M. Choose (M, A_m) to match output per worker and the manufacturing employment share, with $A_s = 1$.

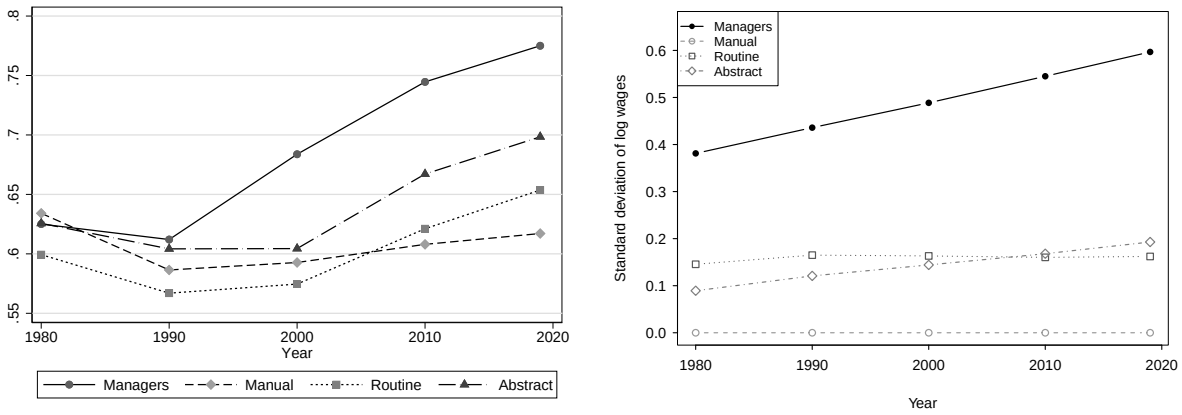
3. Retain these task weights and sector-specific productivities in 2019. Use the separately calibrated 2019 distribution, requirements, and threshold vector $\mathbf{x}_{2019}$, together with observed capital per worker. Choose the 11 task-productivity growth rates $\{m_z, m_0, \ldots, m_9\}$ to fit output growth and within-sector occupational employment shares. The endpoint task productivities are $M_j(2019) = M(1 + m_j)^{39}$.
4. Update the common elasticities jointly with the growth rates to fit the change between the two endpoints. Relative-wage targets discipline (a, γ_h) and the requirement profile at each date. Tables 5 and 6 report the resulting 1980–2019 benchmark.

F Supplementary Tables and Figures

Within-group wage inequality Because our quantitative model has a continuum of skills and discrete occupations, it has implications for wage inequality *within* occupation groups as well. As shown in Figure F.2(a), the standard deviation of log wages rose among managers, abstract workers and routine workers, and remained more or less constant among manual workers.

The model gives rising dispersion among managers, abstract workers and routine workers, and constant dispersion among manual workers. It understates dispersion levels, while the relative magnitudes of the changes differ from the data. Since occupational mean wages do not determine the full within-group distribution, dispersion is a further dimension on which to assess the model. For manual workers, the model implies zero within-occupation dispersion because effective skill is constant, whereas dispersion is positive in the data.

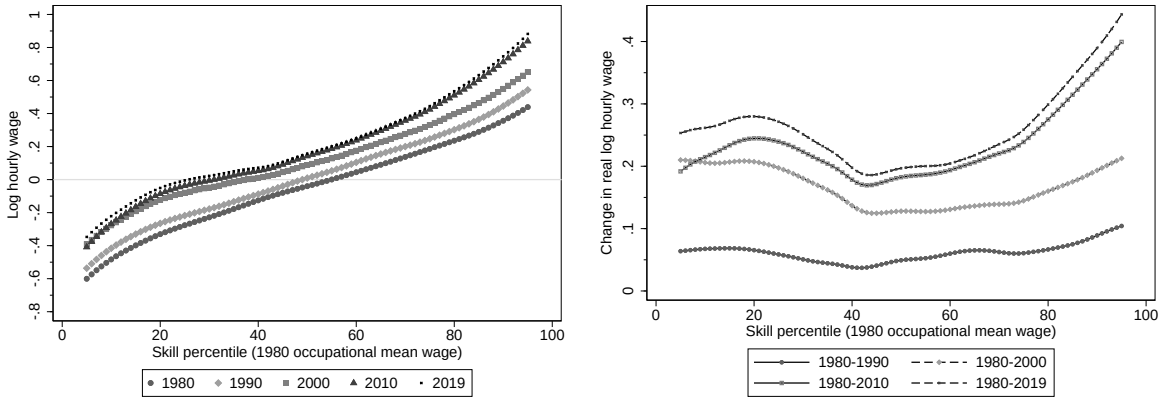
Average size of establishments The managerial margin has a related organizational interpretation. With a constant number of managers per establishment, the faster rise in manufacturing's manager-to-worker ratio implies a faster decline in workers per establishment. Figure F.4(a) shows this pattern in the



(a) Data (b) Model

Fig. F.2: Within-Occupation Wage Inequality
Standard deviation of log wages within the four occupation groups in the data and the model.

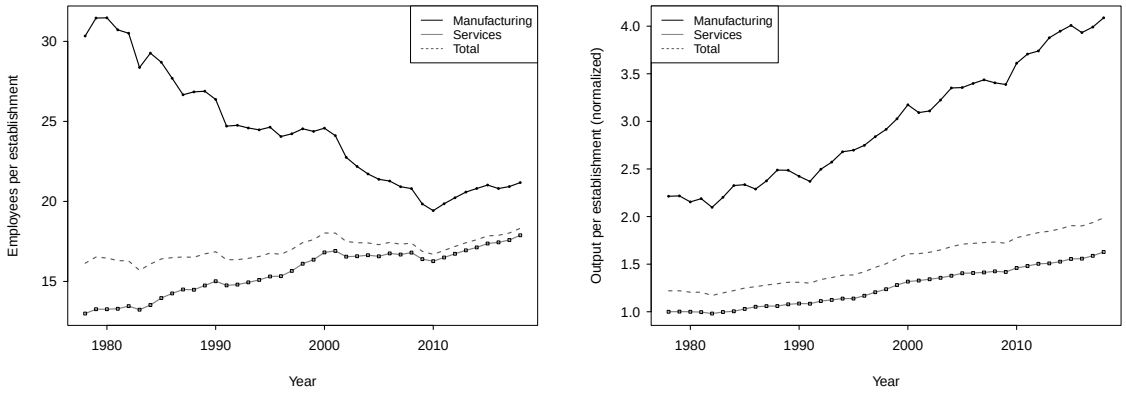
BDS; panel (b) combines BEA value added with establishment counts to show output per establishment. The model does not determine an establishment-size distribution, so this is a supporting comparison conditional on the manager–establishment relationship. Figures F.5 and F.6 report related cross-industry comparisons. In the aggregate series in Figure F.4(a), employment per establishment does not fall alongside the rise in the managerial employment share, indicating that a fixed number of managers per establishment does not capture all organizational changes.



(a) Wage Profiles

(b) Cumulative Wage Growth

Fig. F.3: Occupational Wage Profiles and Growth by Initial Wage Rank, 1980–2019.
Occupations are ranked by their 1980 mean hourly wage, using employment-weighted skill percentiles. Panel (a) shows log mean hourly wages relative to the 1980 mean across percentile bins, for 1980, 1990, 2000, 2010, and 2019. Panel (b) shows cumulative changes from 1980 to each subsequent date, rather than separate decade changes. Occupational means use labor-supply weights; the profiles use the same fixed initial rank and LOWESS bandwidth of 0.4 as the endpoint wage comparison. These are changes across occupation ranks, not longitudinal wage changes for individual workers.



(a) Average No. of Workers

(b) Average Value-added Output

Fig. F.4: Establishment Size by Sector

Source: BDS and BEA Industry Accounts, 1978–2018. Left: Average number of workers per establishment, in aggregate and by sector. The BDS manufacturing group includes mining, utilities, construction and manufacturing; services comprise the remaining reported industries. Right: Real value-added output per establishment, normalized by service output per establishment in 1980. BEA services output includes government; BDS counts cover private establishments.

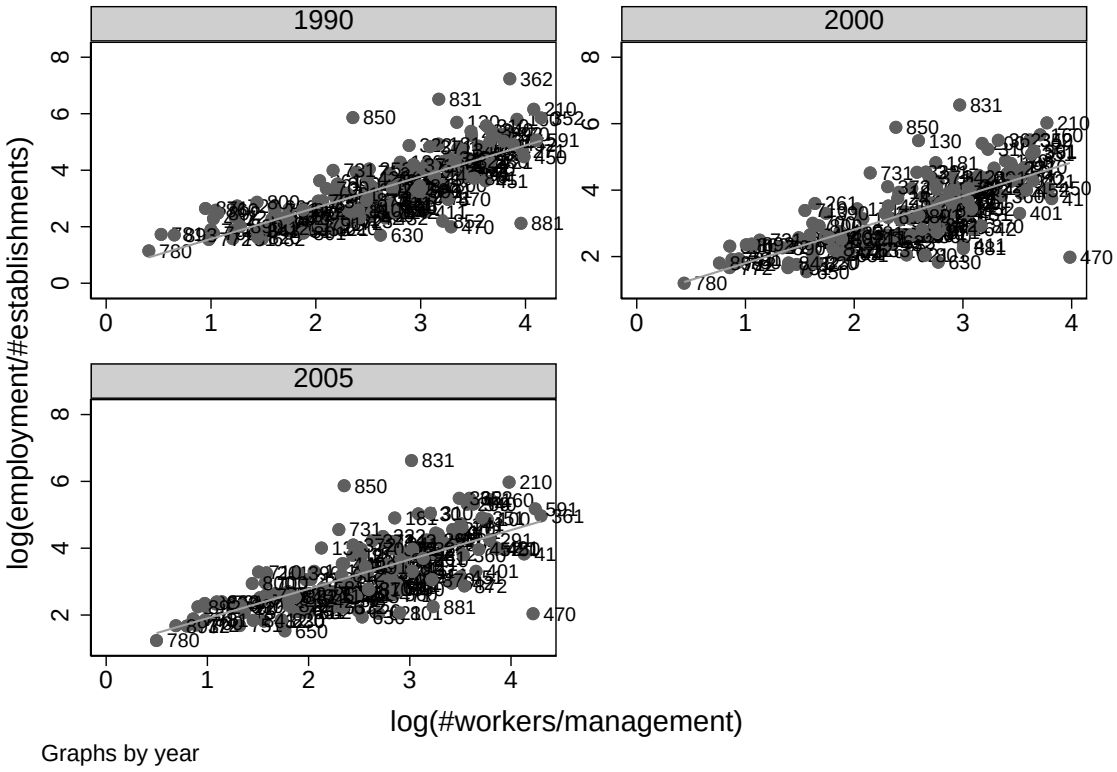


Fig. F.5: Establishment Size and Managerial Span of Control.

Cross-industry comparison of log employment per establishment (vertical axis, SUSB) and log workers per manager (horizontal axis, Census), by year. The sample contains 170 industries. These descriptive associations do not establish a one-to-one mapping between managers and establishments.

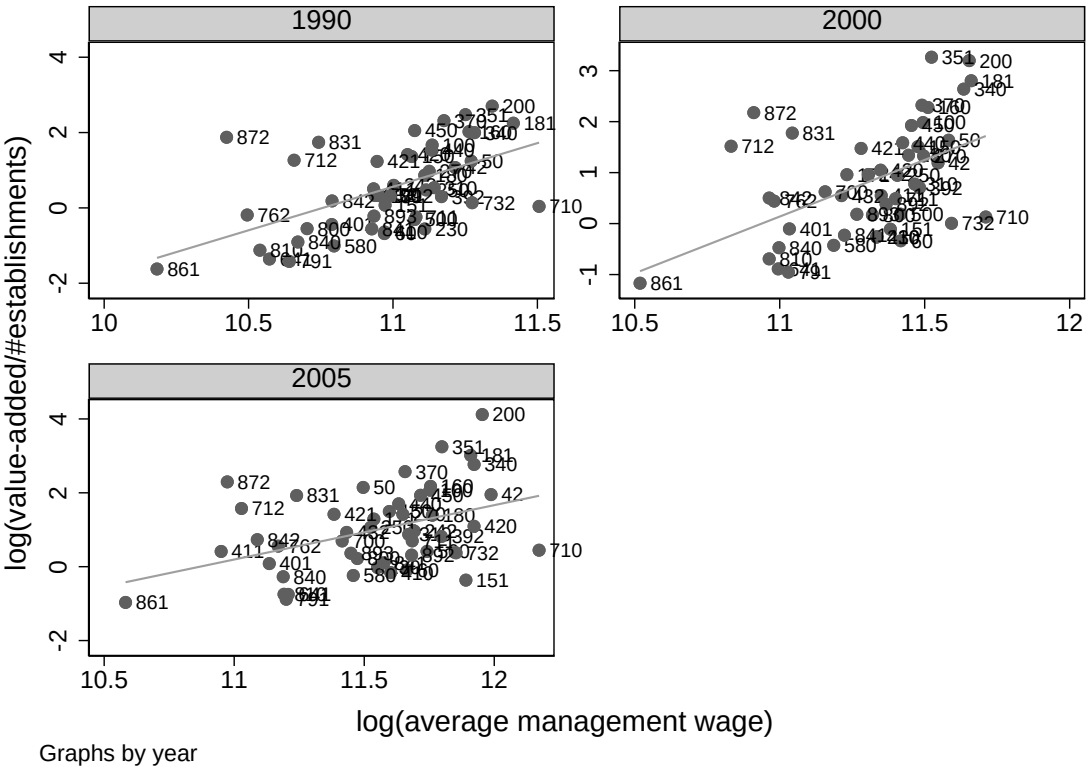


Fig. F.6: Establishment Output and Manager Compensation.
Cross-industry comparison of log value added per establishment (vertical axis, BEA-based measure) and log average managerial wages (horizontal axis, Census), by year. The sample contains 48 industries. The comparison is descriptive.

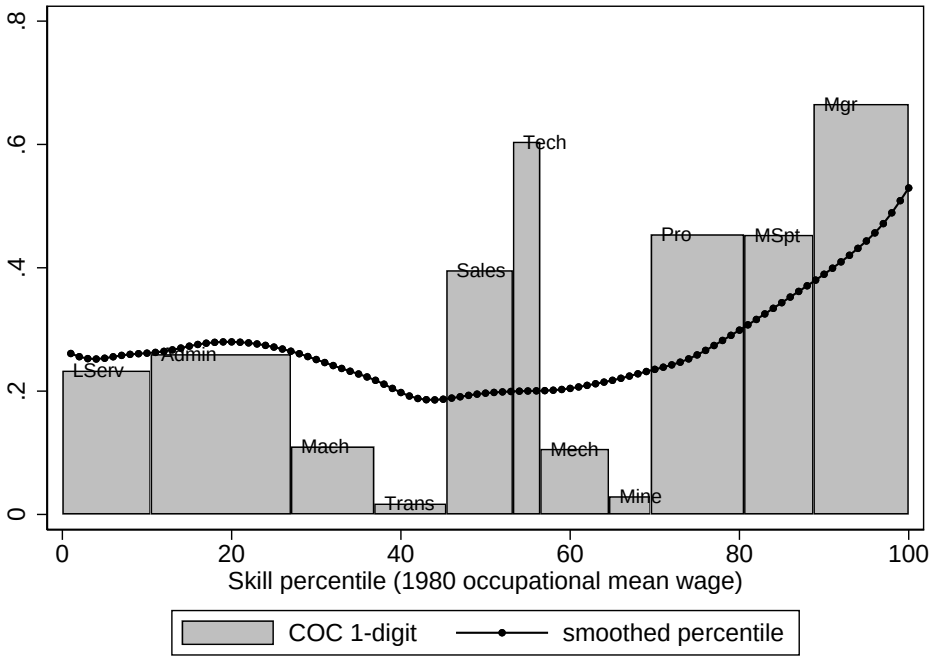


Fig. F.7: Occupational Wage Growth, 1980–2019.

Source: U.S. Census and ACS. Occupations are ranked by their 1980 mean wage; bars represent the 11 broad occupation groups and the line smooths across the 310 detailed occupations. This growth panel complements the wage levels in Figure 1(b). Figure F.3 shows the intermediate cumulative changes along the initial wage ranking.

Ranked by mean wage (except management)	COC Code	Relative Wages 1980	2019
Low Skill Services	400	**0.65**	**0.53**
Middle Skill		**0.90**	**0.76**
Administrative Support	300	0.78	0.65
Machine Operators	700	0.84	0.61
Transportation	800	0.88	0.58
Sales	240	0.94	0.90
Technicians	200	1.04	1.22
Mechanics & Construction	500	1.06	0.76
Miners & Precision Workers	600	1.09	0.72
High Skill		**1.26**	**1.27**
Professionals	40	1.21	1.23
Management Support	20	1.32	1.34
Management	1	**1.39**	**1.74**

(average wage in each year normalized to 1)

Table F.1: Occupational Relative Wages, 1980–2019

Source: 1980 Census and 2019 ACS; occupation definitions follow Appendix A. Mean wages are normalized by the overall mean wage in each year. Broad middle- and high-skill rows summarize the underlying occupation groups. Calibration targets are the seven individual routine wages and the three broad-group means relative to manual wages; the individual high-skill wages are reported for comparison. Employment targets are shown in Figures 9 and 2.

ΔAvg Wage	(1)	(2)	(3)
TFP Growth	-6.082**		-2.895*
	(0.930)		(0.899)
Δ_χ		1.378**	0.889**
		(0.190)	(0.181)
Constant	0.395**	0.111*	0.223**
	(0.039)	(0.040)	(0.044)
R^2	0.755	0.822	0.890

Δ_χ	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
RTI (DOT)	-0.053							
	(0.045)							
R-Manual		-0.109**				-0.064*		-0.053
		(0.024)				(0.020)		(0.029)
NR-Man Pers			0.093*					
			(0.030)					
NR-Cog Anal				0.113**		0.075*	0.087**	0.069*
				(0.026)		(0.024)	(0.024)	(0.028)
ΔCol Share					1.309**		0.638	0.295
					(0.340)		(0.302)	(0.409)
Constant	0.139*	0.139**	0.139**	0.139**	-0.011	0.139**	0.066	0.106
	(0.044)	(0.033)	(0.037)	(0.031)	(0.044)	(0.028)	(0.043)	(0.066)
R^2	0.126	0.527	0.384	0.569	0.407	0.687	0.635	0.698

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$

Table F.2: Effective Skills, Wage Growth, and Task Content, 1980–2019

The first panel relates 1980–2019 log occupational mean-wage growth to annual net task-productivity growth $m_j = (M_j(2019)/M_j(1980))^{1/39} - 1$, in decimal units, and Δ_χ; the second relates Δ_χ to task content and the change in the college share. Both panels use the benchmark calibration and the signed log effective-skill index defined in Section 5, as in the figures. There are eleven occupation groups, using the normalized tail-ability statistic for managers. Regressions are unweighted OLS with heteroskedasticity-robust standard errors (HC1). Task indices are standardized across the eleven groups; the college-share change is in levels. Wages are calibration targets, so their association with the inferred index is not independent validation. The task-content associations provide additional descriptive evidence.

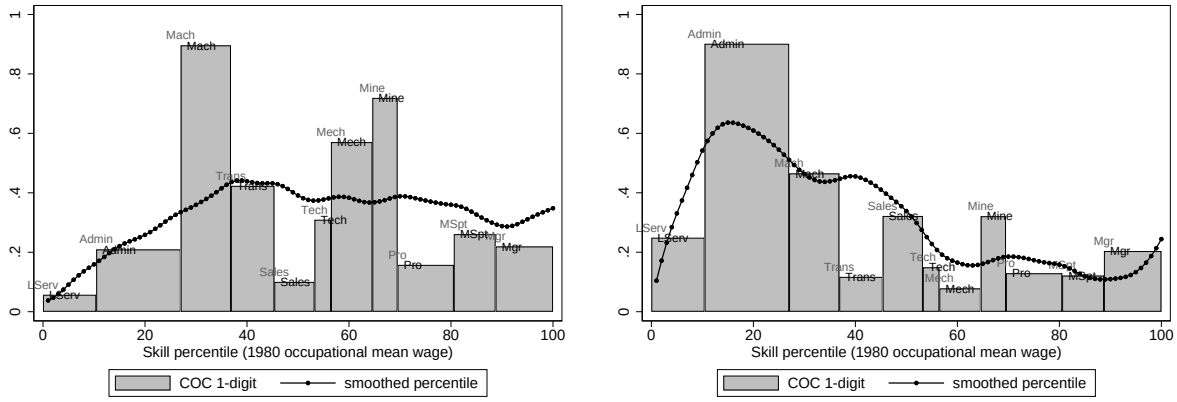
ΔEmp Share	(1)	(2)	(3)
TFP Growth	-11.058**		-15.816**
	(1.821)		(2.024)
Δ_χ		1.343	-1.327**
		(0.719)	(0.331)
Constant	0.177*	-0.178	0.434**
	(0.073)	(0.159)	(0.077)
R^2	0.837	0.262	0.938

TFP Growth	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
RTI (DOT)	0.019							
	(0.009)							
R-Manual		0.026**				0.029	0.023*	0.023*
		(0.005)				(0.017)	(0.009)	(0.007)
NR-Man Pers			-0.024**			0.003		
			(0.004)			(0.015)		
NR-Cog Anal				-0.020*			-0.006	
				(0.007)			(0.009)	
ΔCol Share					-0.278**			-0.072
					(0.078)			(0.098)
Constant	0.015	0.015*	0.015	0.015	0.047**	0.015*	0.015*	0.023
	(0.008)	(0.006)	(0.007)	(0.008)	(0.013)	(0.006)	(0.006)	(0.012)
R^2	0.351	0.660	0.526	0.374	0.390	0.661	0.685	0.675

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$

Table F.3: Task Productivity, Employment, and Task Content, 1980–2019

The first panel relates the 1980–2019 proportional change in occupational employment shares to annual net task-productivity growth m_j, in decimal units, and Δ_χ. The second relates task-productivity growth to task content and changes in the college share. Both panels use the benchmark calibration and eleven occupation groups, with the same index, sample, standardization and unweighted OLS/HC1 conventions as Table F.2. Statistical associations do not identify causal effects.

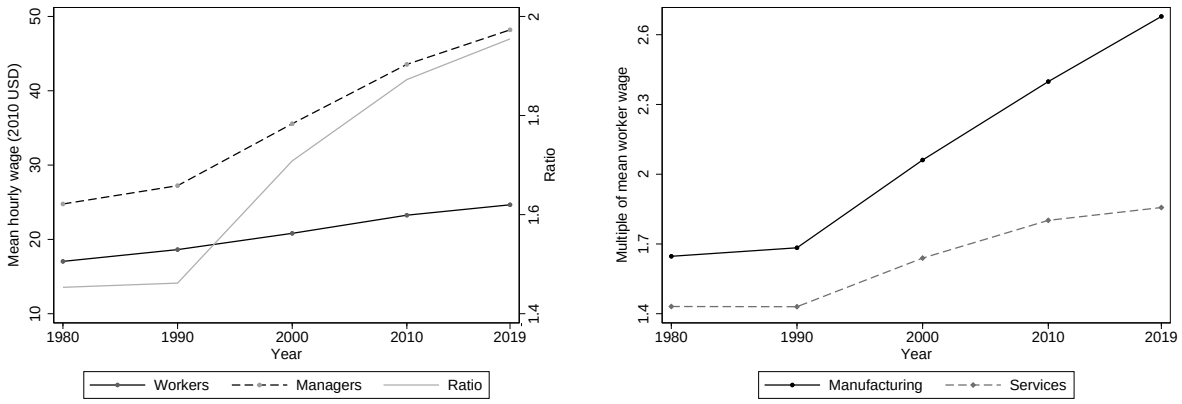


(a) Manufacturing

(b) Routinization

Fig. F.8: Manufacturing Employment Shares and Routine Job Shares

Source: U.S. Census (5%). Left: Manufacturing employment share by occupation-skill percentile in 1980. Right: Share of employment in occupations in the top employment-weighted third of the routine task intensity (RTI) distribution, by skill percentile. The RTI measure follows Autor and Dorn (2013) and uses detailed occupational task requirements from the DOT. Occupations are ranked by their 1980 mean wage for 11 one-digit groups and smoothed across 322 three-digit groups, separately. The x-axis measures employment percentiles. Further details are in Appendix A.

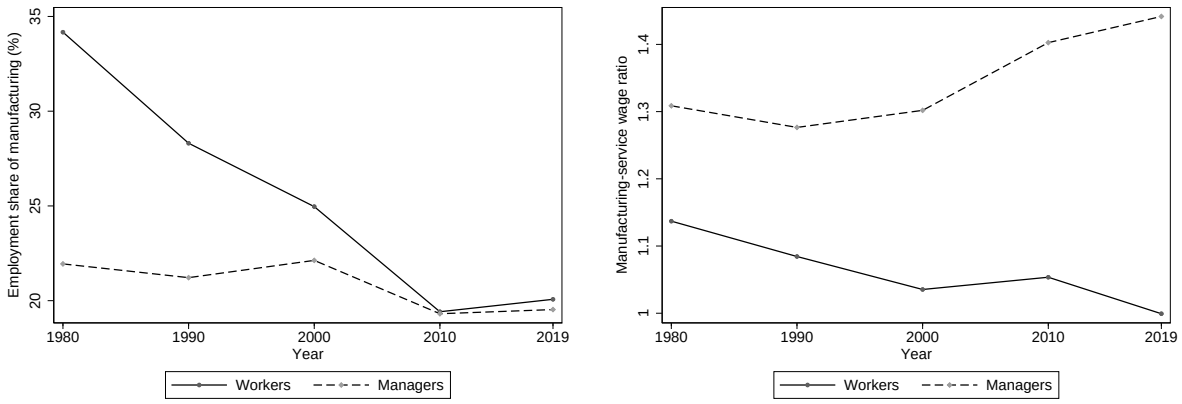


(a) Aggregate Wage Levels and Ratio

(b) Manager Wage by Sector

Fig. F.9: Relative Manager Wages

Source: U.S. Census and ACS. Left: levels and ratio of mean wages of managers and all other workers in aggregate. Right: relative mean wage of managers over all other workers within manufacturing and services. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. See Appendix A for the definition of managers and Figure A.1 for a detailed breakdown of the manager group.

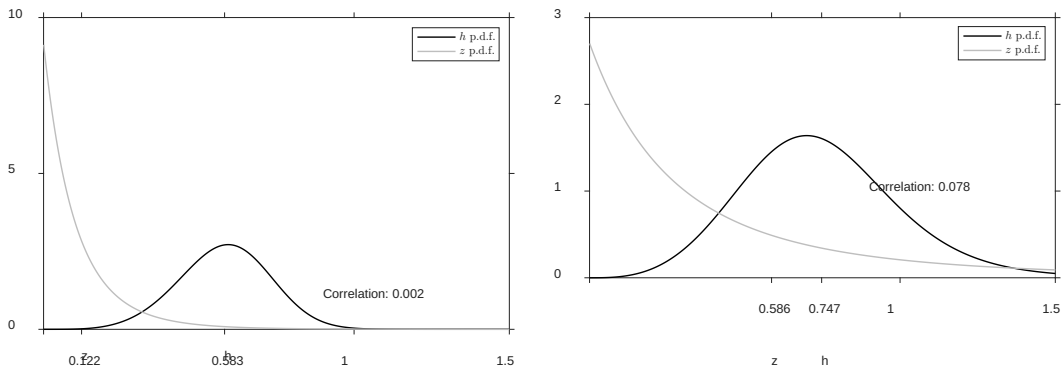


(a) Manufacturing Employment Share

(b) Manufacturing-Services Average Wage Ratio

Fig. F.10: Manufacturing vs. Services by Occupation

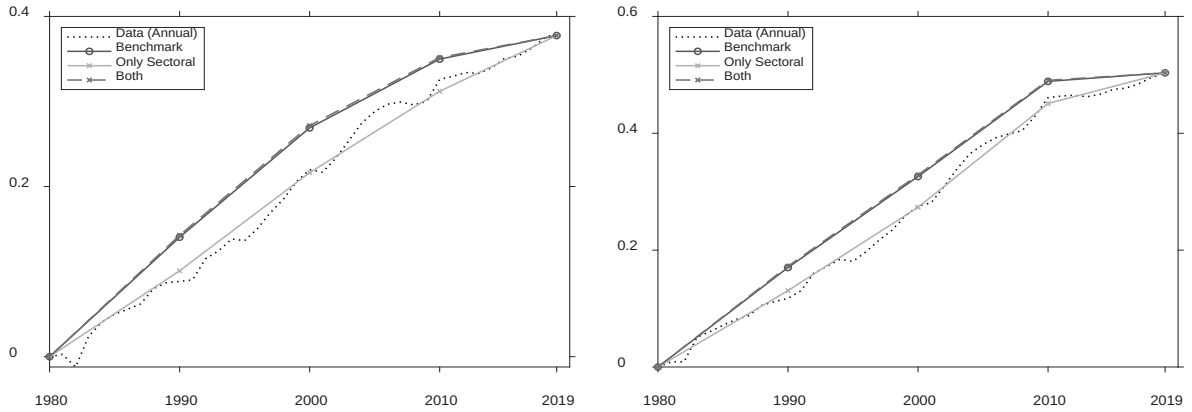
Source: U.S. Census and ACS. Left: manufacturing employment share within the manager occupation group and all other workers. Right: mean wage of manufacturing employment relative to services employment within the manager occupation group and all other workers. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. See Appendix A for the definition of managers and Figure A.1 for a detailed breakdown of the manager group.



(a) 1980 (b) 2019

Fig. F.11: Calibrated Skill Distribution

We use a type IV bivariate Pareto distribution to model the distribution over worker and manager skills (h, z). The figure depicts the marginal distributions of each skill, and also their mean below the x-axis.

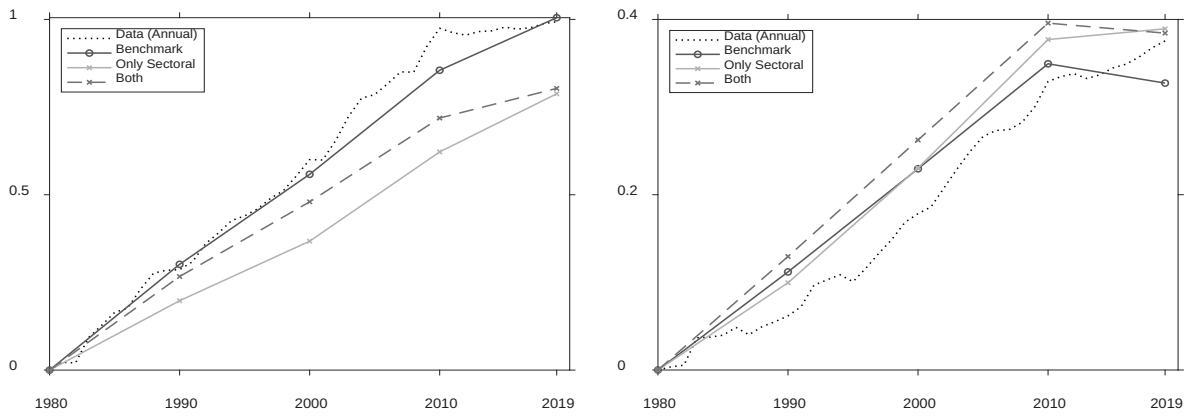


(a) Log TFP Growth

(b) Log GDP per Worker Growth

Fig. F.12: Aggregate Output and TFP Growth

Data: NIPA. Log 1980 levels are normalized to 0, so the slopes of the lines are the growth rates.

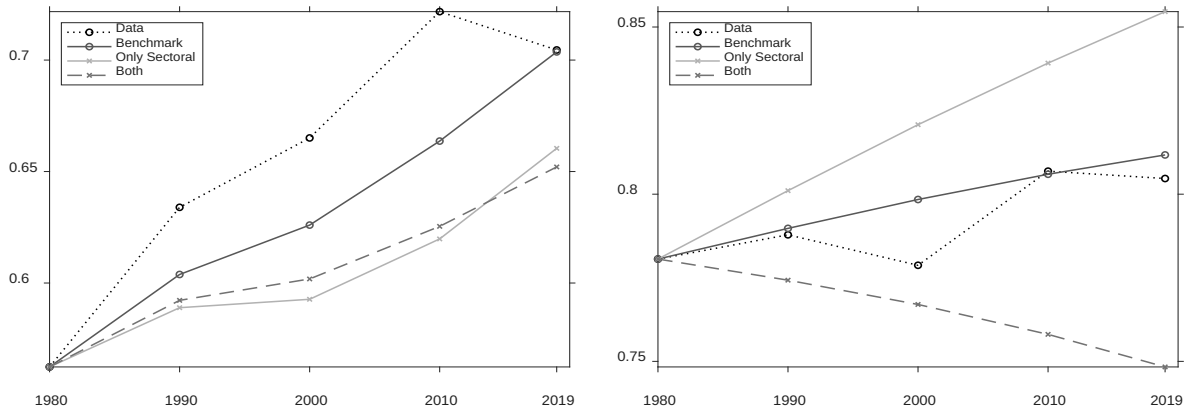


(a) Log GDP per worker, Manufacturing

(b) Log GDP per worker, Services

Fig. F.13: Benchmark vs. Counterfactuals, GDP per Worker

Data: NIPA. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. Log 1980 levels are normalized to 0, so the slopes of the lines are the growth rates.



(a) Services Employment Share, Routine Jobs

(b) Services Employment Share, Managers

Fig. F.14: Benchmark vs. Counterfactuals, Structural Change
Vertical axes are the fractions of routine jobs (left) and managers (right) working in services.

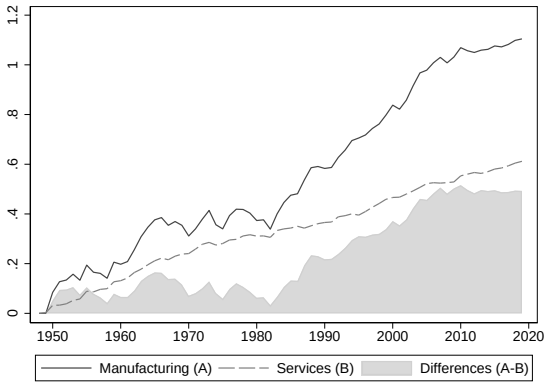
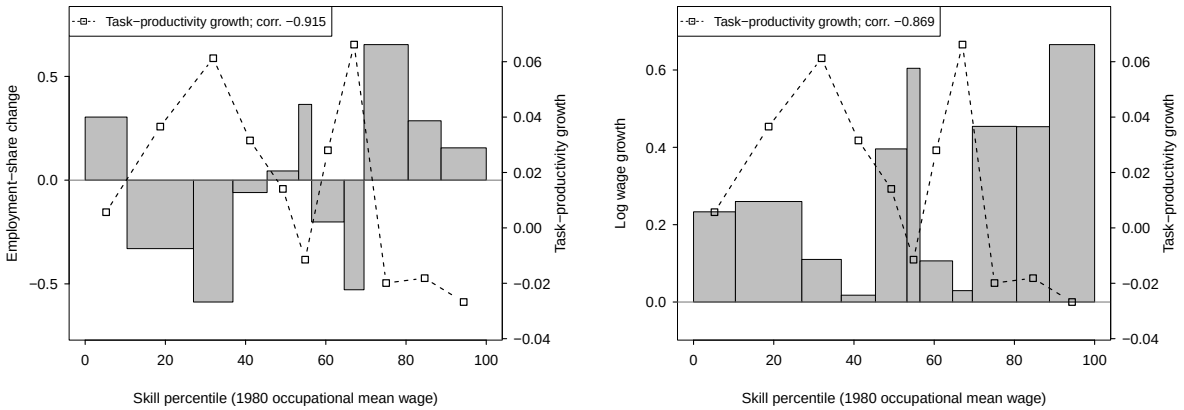


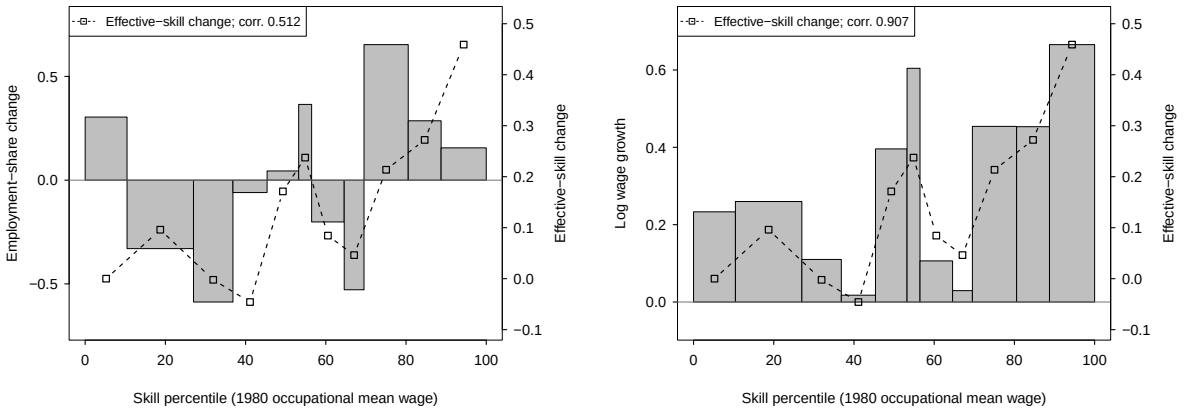
Fig. F.15: Log Sectoral TFP, 1948–2019.

Source: BEA NIPA accounts. Sectoral output and capital are computed via cyclical expansion from the industry accounts as in Herrendorf et al. (2013). Employment is based on full-time equivalent number of persons in production in NIPA Table 6. Within each sector, TFP is measured as the Solow residual using its average capital income share (0.332 in manufacturing and 0.365 in services); log-TFPs are normalized to 0 in 1948.



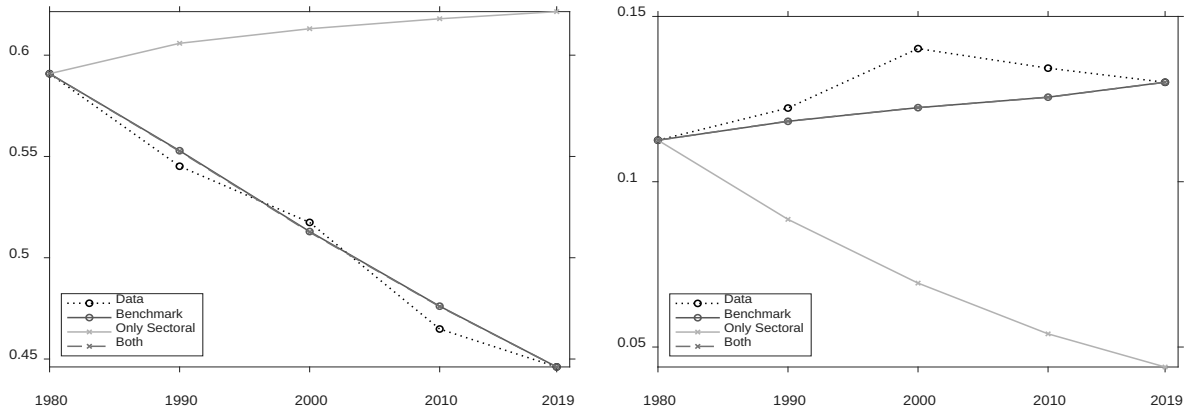
(a) Employment (b) Wages

Fig. F.16: Task Productivity Growth, Employment Change, and Wage Growth.
The dashed line shows calibrated annual task-productivity growth on the right axis in both panels. The bars show the proportional change in occupational employment shares in panel (a) and the log change in occupational mean wages in panel (b), on the left axis. Occupations follow their 1980 wage ranking; the plotted correlations describe associations across occupations.



(a) Employment (b) Wages

Fig. F.17: Effective-Skill Changes, Employment Change, and Wage Growth.
The dashed line shows Δ_χ, the signed log change in effective skill relative to mean skill, on the right axis in both panels. The bars show the proportional change in occupational employment shares in panel (a) and the log change in occupational mean wages in panel (b), on the left axis. Occupations follow their 1980 wage ranking; the plotted correlations describe associations across occupations.



(a) Aggregate Employment Share, Routine Jobs

(b) Aggregate Employment Share, Managers

Fig. F.18: Benchmark vs. Counterfactuals, Polarization

Vertical axes are the aggregate employment shares of routine jobs (left) and managers (right).

F.1 Task-Content Associations across Occupation Groups

The main task-content comparisons use eleven broad occupation groups. Two groups warrant separate checks: the manual effective-skill index is normalized to zero, and the managerial index summarizes the joint skill distribution rather than a worker-task requirement. Table F.4 repeats the correlations among workers only, among nonmanual workers, and after omitting each group in turn. It uses the same saved occupational aggregates as Figures 16 and 17, with no re-estimation of the model.

		Effective-skill index Δ_χ			Task TFP
Sample	N	Wage growth	Analytical	ΔCollege	Routine-manual
All groups	11	0.907	0.755	0.638	0.812
Workers only	10	0.888	0.881	0.779	0.806
Nonmanual workers	9	0.925	0.865	0.793	0.812
Omit one group: minimum	10	0.888	0.699	0.589	0.754
Omit one group: maximum	10	0.926	0.881	0.779	0.883

Table F.4: Task-Content Associations: Sensitivity to Occupation Groups

Unweighted Pearson correlations. The three effective-skill columns relate Δ_χ to log mean-wage growth, nonroutine-cognitive analytical content, and the change in the college share. The final column relates annual task-productivity growth to routine-manual content. Changes refer to 1980–2019. The last two rows give the minimum and maximum correlation over all eleven samples formed by omitting one group, each with ten observations; the omitted group can differ across columns. Workers only excludes managers; nonmanual workers also excludes the manual group. Wage growth is partly targeted by the calibration; task content and college-share changes are not targets.

The effective-skill correlations with analytical content and the college-share change rise to 0.881 and 0.779 when managers are excluded, and remain positive in every sample. The productivity–routine-manual correlation ranges from 0.754 to 0.883 when one group is omitted. These associations therefore persist without the managerial entry and do not depend on a single occupation group. The checks assess the influence of individual groups on the descriptive relationships; they do not address sensitivity to the calibrated parameters or establish causal effects.

F.2 AI Exposure and Scenario Comparison

The comparison in Section 6.1 changes the exposure measure, skill-scenario intercept and common GDP-normalization level, holding the joint skill-index construction fixed. Let E_j denote standardized exposure and $q_j = \log(1 - \chi_j/\bar{h})$ for nonmanual worker tasks, where $\bar{h} = \mathbb{E}h$. The 2030 scenario sets $q_{j,2030} = q_{j,2019} - (11/39)(x + bE_j)$. The loading b is the ratio of the standard deviation of historical changes in q to the standard deviation of exposure across the eleven groups. The skill intercept x specifies the common component of index changes, separately from their dispersion across occupations. We set $x = 0.470$ for broad AI and $x = 0.390$ for LLM exposure. These levels are assumptions of the respective scenarios, not estimates from exposure or restrictions implied by matching GDP. A common task-productivity level provides the separate GDP normalization. The manual task has $q_0 = 0$ throughout, with neither an intercept nor an exposure displacement, and constant effective skill per worker. Its zero index is a normalization, rather than the log of physical manual input divided by population mean skill.

For the managerial entry, use $q_z = \log(1 - C_z/\bar{h})$, where $C_z = (1 + \bar{h}^{1/\gamma_h})^{\gamma_z}\Gamma(1+\gamma_z)\Gamma(a-\gamma_z)/\Gamma(a+1) = \mathbb{E}[z \mid h \geq \bar{h}]/a$. Here Γ denotes the gamma function. The point-conditional managerial mean is $\bar{z}^{\text{ref}} = \mathbb{E}[z \mid h = \bar{h}] = (a - \gamma_z)C_z$. Its ratio to effective worker skill in task j satisfies

$$\log \frac{\bar{z}^{\text{ref}}}{\bar{h} - \chi_j} = \log \frac{(a - \gamma_z)C_z}{\bar{h}} - \log \frac{\bar{h} - \chi_j}{\bar{h}}.$$

Thus q_z is a coordinate of the joint distribution, not this relative-ability ratio or a selected-manager mean. The historical regressions and projection use the same coordinate and sign. In 2030, (a, γ_h) jointly match its displaced value and the extrapolated mean worker skill; the changing requirements χ_j remain in the worker entries. Thus exposure changes the calibrated joint skill distribution.

The continuation extrapolates the 1980–2019 changes in unconditional worker and managerial mean skills and in worker q_j linearly. Both scenarios retain the historical relative task-productivity growth rates and share a common capital path. After 2030, worker q_j changes by $-(10/39)(x + bE_j)$ per decade; unconditional managerial mean skill repeats its 2019–2030 increment in each later decade. These rules specify the conditional paths through 2050. Let $\widetilde{Y}_{2030}$ denote scenario output before the GDP adjustment. A common multiplier $\lambda = Y_{2030}^{\text{continuation}}/\widetilde{Y}_{2030}$ scales all eleven task productivities, including management, in both sectors: $\lambda = 1.052078$ for broad AI and 1.047604 for LLM exposure. The adjustment is reached by 2030 and retained as a level thereafter; the historical growth rates then continue. With fixed capital, the common capital share and homothetic production, this normalization scales output and all earnings levels equally without changing the skill construction, allocations or relative wages.

Group	Raw exposure		Standardized		Rank	
	AIOI	β	AIOI	β	AIOI	β
Low Skill Services	0.641	0.165	-1.114	-0.736	9	7
Administrative Support	0.699	0.486	1.064	1.243	2	1
Machine Operators	0.652	0.075	-0.725	-1.291	8	11
Transportation	0.633	0.115	-1.418	-1.043	11	9
Sales	0.679	0.436	0.320	0.935	6	3
Technicians	0.690	0.396	0.734	0.689	4	4
Mechanics and Construction	0.641	0.096	-1.140	-1.156	10	10
Miners and Precision	0.657	0.150	-0.524	-0.829	7	8
Professionals	0.695	0.395	0.916	0.682	3	5
Management Support	0.705	0.456	1.299	1.058	1	2
Model managers	0.686	0.357	0.588	0.447	5	6

Table F.5: Broad-AI and LLM Exposure by Model Group.

AIOI is the Felten et al. measure; β is the human rating of Eloundou et al. Raw indices have different units. Standardized scores subtract the equal-group mean and divide by the sample standard deviation across all eleven groups, before setting the manual loading to zero; rank 1 denotes greatest exposure. Professionals and management support are abstract-worker groups; model managers include self-employed people. Both measures use 2010 labor-supply weights. The LLM means use matched ratings after the official O*NET 2010–2019 crosswalk, with equal weights across unique rated descendant occupations within each Standard Occupational Classification (SOC) code and broad-code and occupation-family matching for residual codes, followed by aggregation to the model groups. Unmatched scores remain missing. Coverage in table order is 100.00, 98.31, 98.57, 99.67, 100.00, 100.00, 99.58, 100.00, 99.74, 100.00 and 99.90 percent, or 99.60 percent overall. On common matched support the largest shift in an AIOI raw group mean is 0.000078; Pearson correlation changes from 0.937885 to 0.937652 and Spearman remains 0.845455.

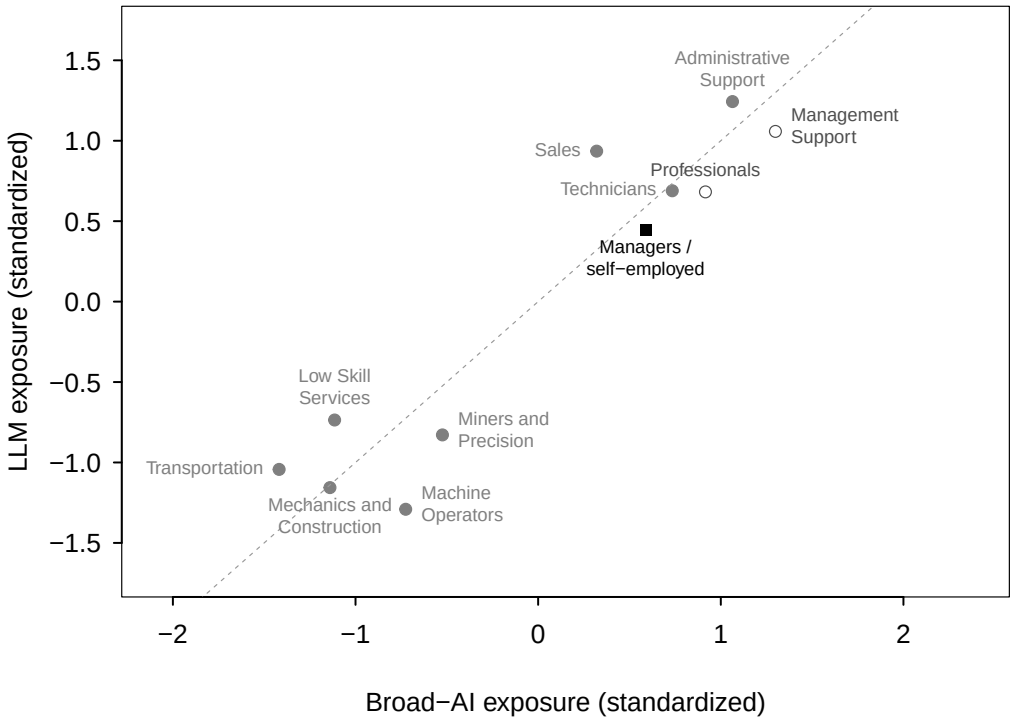


Fig. F.19: Comparison of Standardized AI Exposure Measures.

Eleven model groups, including the separate manager group, before setting the manual loading to zero. The dashed line is $y = x$; equal standardized scales do not imply equal capability units. The group-level Pearson and Spearman correlations are 0.937885 and 0.845455; positive affine standardization leaves both unchanged. On a common sample of 471 Census-2010 occupations, before manager/self-employed reassignment, unweighted Pearson and Spearman correlations are 0.840048 and 0.849401. Using normalized 2010 labor-supply weights gives Pearson 0.837980 and weighted rank correlation 0.839163. Weighted Pearson is weighted covariance divided by weighted standard deviations; the rank statistic applies this formula to common-sample average-rank vectors, one convention for weighted rank correlation. Each occupation enters once. Detailed matched coverage is 99.60 percent; the by-group figures are in Table F.5.

Date	Path	GDP	Manual	Routine	Abstract	Manager
A. GDP (model units) and employment shares (percent)						
2030	Continuation	1.7132	13.28	39.02	33.76	13.94
2030	Broad AI (AIOI)	1.7132	13.30	40.54	32.13	14.03
2030	LLM exposure (β)	1.7132	13.14	39.03	33.59	14.24
2050	Continuation	1.6067	11.45	30.64	42.70	15.21
2050	Broad AI (AIOI)	1.4842	11.64	33.43	40.16	14.77
2050	LLM exposure (β)	1.4979	11.33	31.34	41.84	15.50
B. Occupational mean earnings relative to economy-wide mean						
2030	Continuation	—	0.530	0.727	1.137	1.879
2030	Broad AI (AIOI)	—	0.518	0.706	1.206	1.834
2030	LLM exposure (β)	—	0.525	0.730	1.145	1.835
2050	Continuation	—	0.552	0.685	0.951	2.109
2050	Broad AI (AIOI)	—	0.516	0.651	1.054	2.023
2050	LLM exposure (β)	—	0.533	0.686	0.988	2.007

Table F.6: Conditional AI Scenario Outcomes, 2030 and 2050.

Scenario construction follows the mapping and continuation rules described above. GDP is aggregate model output. Employment shares refer to persons, while relative mean earnings equal each group's wage-bill share divided by its employment share. Routine combines the seven middle-skill worker occupations; abstract combines professionals and management support. Both exposure scenarios match continuation GDP in 2030, not in 2050. In 2050, LLM minus broad-AI employment-share differences are $-0.31, -2.09, +1.68, +0.73$ percentage points for manual, routine, abstract and managers; relative mean-earnings differences are $+3.22, +5.25, -6.46, -0.79$ log points.

F.3 Polarization or Structural Change?

Lemma 3 also raises the converse question: how much occupational polarization can arise from sectoral reallocation alone? We isolate this composition effect while holding occupational shares within sectors fixed.

Hold within-sector occupational shares and relative average earnings fixed. Under this conditional calculation, a rise in manufacturing productivity A_m raises $\log \kappa$ at rate $1 - \epsilon$. Thus the direct composition contribution is

$$\left.\frac{d \log L(j)}{d \log A_m}\right|_{\text{composition}} = (1 - \epsilon)\left[\frac{L_s(j)}{L(j)} L_m - \frac{L_m(j)}{L(j)} L_s\right]. \tag{82}$$

It is negative for occupations with a higher within-sector employment share in manufacturing. This is the occupation-specific version of the composition term in (78); it is not the full equilibrium derivative when assignment and occupational selection respond.

For a finite comparison, let $s_{mR,1980}$ and $s_{sR,1980}$ denote routine employment shares within manufacturing and services. Holding those compositions fixed gives

$$\Delta L_R^{\text{composition}} = (L_{m,2019} - L_{m,1980})(s_{mR,1980} - s_{sR,1980}).$$

The model's 1980 occupational allocations give $s_{mR,1980} = 25.86/32.79$ and $s_{sR,1980} = 33.23/(100 - 32.79)$. Using the manufacturing shares 32.79 and 19.88 percent from Table 10, the composition contribution is approximately -3.80 percentage points. Aggregate routine shares fall from 59.08 to 44.61 percent, or about 14.5 percentage points. Thus this fixed-composition calculation accounts for about 26 percent of the change. This is an accounting comparison, not a bound on the equilibrium effect of sector-specific technology.